Adaptive Radio Spectrum Allocation through Mid-Term Reconfigurations for Cellular Communications Systems

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Abstract. A fundamental problem in cellular communications systems is the adaptation of the frequency allocation to the traffic fluctuations. Traffic adaptation may be achieved by a computationally efficient solution to the following problem: "Given the radio spectrum available, the cell structure, the current traffic conditions, and an already established allocation of frequencies to cells, find a new optimal allocation that is more suitable for handling the current traffic conditions, taking into account the previous allocation and subject to the restrictions arising from the interference conditions". In this paper, this problem is formally defined, optimally formulated and solved in a computationally efficient manner. The results provided demonstrate a significant increase in the efficiency with which the available spectrum is utilised, using as a basis for comparison legacy frequency allocation schemes.

Keywords. GSM, UMTS, frequency allocation, 0-1 quadratic programming, weighted matching, DCA

1. INTRODUCTION

Legacy and future cellular communications systems (e.g., the future versions of the Global System for Mobile communications-GSM [1], or the Universal Mobile Telecommunications System - UMTS [2,3,4]) should be able to cope with heavy traffic loads. Hence, efficient utilisation of the scarcely available radio spectrum becomes a fundamental problem. Currently, cellular systems use static allocation schemes (Fixed Channel Allocation - FCA). The allocation of channels to cells is determined in the design phase of the system and is occasionally changed due to the accumulated experience. The allocation is determined by the traffic expected in each cell and it usually takes into account peak hour traffic data. Two major improvements may be envisaged for future cellular systems, both motivated by the fact that the traffic load is time-variant. First, the (as fast as possible) reconfiguration of the frequency allocation in various time zones of the day (mid-term reconfigurations) so as to adapt to (expected or unpredicted) traffic variations (traffic adaptive spectrum allocation [5]). Second, the application of a Dynamic Channel Allocation (DCA) strategy [6,7] that may constantly (or at least more frequently) rearrange the frequency allocation.

In this paper we present an essential component of a radio spectrum management scheme that is suitable for performing fast (and hence, applicable in real-time) frequency allocation reconfigurations in various time-zones of the day (mid-term reconfigurations). Starting from a *basic* version of the spectrum allocation problem (that is targeted to fixed traffic loads), we define, optimally formulate and solve an *extension* that is suitable for handling time variant loads. In general, we envisage a radio spectrum management scheme that will comprise mid-term reconfiguration algorithms for handling major traffic variations (expected to be encountered among different time-zones of the day), and DCA algorithms for handling short-term or local traffic fluctuations (expected to be encountered within a time-zone in the day). The detailed presentation of the overall scheme is not covered in this paper. Instead we analyse the mid-term reconfiguration component and provide an initial set of promising results from the combined scheme (mid-term reconfiguration and DCA components).

The local channel allocation reconfiguration (LCAR) DCA scheme, which has been introduced and analysed in [8], is considered in this paper. The basic idea of the scheme is to face channel unavailability in a particular cell through a set of easy to find and impose channel reallocations. In other words, the main focus in this paper is handling a major traffic variation that is delimited by two load vectors (e.g., L_1 and L_2) that characterise the traffic conditions in respective time-zones of the day. Moreover, we apply the DCA scheme to these load vectors [8] and obtain some promising results. The combination of these tools (mid-term reconfiguration and DCA components) provides also the means for handling the transition from one load vector to the other (i.e., handling the intermediate load vectors that occur in the transition from L_1 and L_2).

Some basic assumptions in this study are the following. The available spectrum is split to traffic channels according to an FDMA/TDMA multiplexing scheme. When a carrier frequency is allocated to a base station a collection of *N* channels, multiplexed on the carrier, becomes available for serving the ongoing and the new calls of the respective cell. The available carriers are dynamically assigned to base stations, according to the traffic demand in their cells. Spectrum (carrier) reallocations are triggered by Quality of Service (QoS) degradations in cells of the network. Therefore, per lost call or per lost handover frequency reallocations are not considered in this paper. QoS may be expressed as new call blocking and active call dropping probability. Two base stations may utilise the same carrier frequency if the distance between them is at least equal to a minimum *frequency reuse distance*, which guarantees an acceptable level of co-channel interference. For notation simplicity mainly, in this paper we will consider a fixed reuse distance, even though our schemes may be readily extended (e.g., by means of the techniques in [9]) to more general structures of the compatibility matrix (e.g., those considered in [10]).

The pursuit of a scheme that performs fast frequency allocation reconfigurations may start from a *basic* version of the traffic adaptive spectrum allocation problem, which expresses our aim to find the best possible allocation of frequencies to cells for a fixed (versus time) distribution of the traffic load. The problem (*aggregate frequency allocation* - AFA) may be generally stated as follows: “Given the cell coverage, the traffic load in each cell, the frequency reuse distance constraints, and the set of available frequencies, find the best allocation of frequencies to cells”. In this paper the quality criterion for this problem is the maximisation of the traffic that is carried by the system. However, our schemes can readily be combined with other, perhaps more elaborate or complex, criteria. The main advantage of the AFA problem is that it aims at handling a fixed traffic load, which may be assumed to characterise the (new) traffic conditions in a particular time-zone of the day. As discussed in the next paragraph, an *extension* to this problem may yield a more suitable formulation for handling time variant loads.

An additional objective when handling time-variant traffic loads through successive reconfigurations of the frequency allocation pattern may be the acquisition of each new allocation by minimally disturbing the existing, already established, one. This goal is complementary to the maximisation of the total carried traffic. The practical meaning of the new requirement is that the network should be charged with the minimum possible management effort. Hence, every new allocation should be implemented by performing the minimum number of alterations in the already existing allocation of frequencies to cells. This is an important objective for handling unpredicted traffic variations. The *extended* version of the traffic adaptive spectrum allocation problem (*total reconfiguration of frequency allocation* - TRFA) may be generally described as follows: “Given the cell coverage, the (new) traffic load in each cell, the frequency reuse distance constraints, the set of available frequencies, and an existing allocation of frequencies to cells, find a new allocation of frequencies to cells that maximises the total carried traffic, by performing the minimum number of alterations on the already established allocation”.

Regarding the relation to previous work, and adopting the classification introduced in [7], the AFA and TRFA schemes fall into the *centralised channel allocation* category. It is assumed that a controller gathers information and allocates the frequencies of a central pool for use in the cells. When a change in the traffic conditions is sensed a new allocation is pursued. The main novelty in our schemes, with respect to the relevant ones in [7], is that successive reconfigurations have as an additional aim the minimisation of the alterations that are required on the existing allocation, being at the same time optimal with respect to more classical quality criteria (e.g., minimisation of blocking probability). Moreover, the overall concept of totally reconfiguring the frequency allocation (if designated by the traffic variation) in the mid-term time domain, forms an interesting alternative for radio spectrum management, which has not gathered as much attention in the literature. The total reconfiguration concept incorporates some of the advantages of FCA, while the mid-term concept reduces the risk of oscillations that would result from a more dynamic scheme. The TRFA problem defined above is a more general case of the flexible channel allocation (FLCA) scheme [7,11]. According to FLCA each cell is assigned a set of fixed channels that suffices under light traffic loads. The flexible channels are assigned to those cells whose channels have become inadequate for serving the increasing load.

The TRFA scheme is more general as it does not rely on the fixed allocation of a portion of the available radio spectrum (even though it can be readily expanded to co-operate with an FLCA scheme). This enables handling of a wider set of traffic conditions.

As already stated the LCAR scheme was introduced and analysed in [8] and classified in the *cell-based distributed DCA* category (e.g., [12,13]) adopting again the classification in [7].

In this paper we formally state, in section 2, and provide solutions, in sections 3 and 4, the aforementioned (AFA and TRFA) problems. As both problems fall in the *NP*-complete [14,15] category, sections 3 and 4 comprise also computationally efficient (and hence, applicable in real-time) solutions. In section 5 numerical results will be presented and in section 6 concluding remarks will be drawn. Furthermore, in section 5 the AFA and TRFA problems will be applied in conjunction with the LCAR DCA scheme.

2. PROBLEM STATEMENT

The cell layout of the system will be represented by a graph $G(V,E)$. The set of nodes $V = \left\{u_1, u_2, \ldots, u_{|V|}\right\}$, corresponds to the cells of the system. Each edge of the set E connects neighbouring cells. The basic version of a traffic adaptive radio spectrum allocation problem may be stated as follows:

> *Problem 1 [Aggregate Frequency Allocation - AFA].* Given a collection of carrier frequencies F, a graph $G(V,E)$, which describes the cell coverage of an area, the number of channels per carrier frequencies *N*, the minimum distance *d* between cells which are allowed to use the same frequency, a vector $B = \left\{b_1, b_2, \ldots, b_{|V|}\right\}$ representing the offered load in the system (in Erlangs), find an allocation A of frequencies to cells (where $A = \left\{A_1, A_2, \ldots, A_{|V|}\right\}$, and $A_i \subseteq F$ ($i = 1,\ldots,|V|$) is the set of frequencies allocated to cell u_i) that minimises a cost function $C(B,A)$, subject to the restriction $A_i \cap A_j = \varnothing$ if $d(u_i,u_j) < d$ (where function $d\left(u_i,u_j\right)$ provides the distance among the cells u_i and u_j.

As already stated in the introductory section the quality criterion considered is associated with the maximisation of the traffic carried by the system. Hence, the cost function should be associated with the minimisation of the overall blocking probability, which may be expressed as follows.

$$C(B,A) = \sum_{i=1}^{|V|} w_i \cdot B(b_i, |A_i| \cdot N) \tag{1}$$

$B(b_i, k)$ is the blocking probability in cell-u_i, given the traffic load b_i when k channels are available in the cell. Load b_i aggregates new call and handover requests, following the assumptions in [16]. The probability $B(b_i, k)$ is given by the Erlang-B formula:

$$B(b_i, k) = \frac{b_i^k / k!}{\sum_{l=0}^{k} b_i^l / l!} \tag{2}$$

The weights w_i are usually chosen proportional to the load that is offered per cell. Hence,

$$w_i = \frac{b_i}{\sum_{l=1}^{|V|} b_l} \tag{3}$$

The problem above is effective in handling a certain traffic condition, that is, a given (time invariant) traffic load (which may be the traffic load to be faced by the system within a time-zone in the day). In this respect, a computationally efficient algorithm may be part of a scheme that reconfigures, in a real-time manner the frequency allocation so as to adapt to traffic variations. Nevertheless, in the rest of this section we will specify an extension to this problem that results in a scheme that is more suitable for handling changes in traffic conditions with time.

We denote by A the allocation of frequencies to cells, that is established throughout the network at a certain point in time. This allocation designates the expected Quality of Service (QoS levels) in each cell of the system. Traffic variations cause QoS degradations, and hence, a reconfiguration of the frequency allocation is necessitated. Let $\tilde{B} = \{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_{|V|}\}$ represent the new load vector. Through the reconfiguration mechanism a new allocation of frequencies to cells $\tilde{A} = \left\{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_{|V|}\right\}$ should be imposed. Set $\tilde{A}_i$ comprises the frequencies allocated to cell u_i in allocation $\tilde{A}$. This allocation should possess the following properties. First, it should be compliant to the frequency reuse distance constraints. Second, it should improve the cost function value, that is $C(\tilde{B}, \tilde{A})$ should be minimised. Third, the already established allocation should be taken into account. That is, $\tilde{A}$ should be obtained by performing the minimum possible number of alterations in A. The practical meaning of this requirement is to adapt to the traffic variation as soon as possible, and to avoid having to perform massive network originated handovers. The resulting problem may be stated as follows.

> *Problem 2 [Total Reconfiguration of Frequency Allocation - TRFA].* Given a collection of frequencies F, a graph $G(V, E)$, which describes the cell coverage of an area, the number of channels per carrier frequency *N*, the minimum distance *d* between cells which are allowed to use the same frequency, a vector $\tilde{B} = \left\{\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_{|V|}\right\}$ representing the offered load (in Erlangs) in the system, an allocation A of frequencies to cells (where $A = \left\{A_1, A_2, \ldots, A_{|V|}\right\}$, $A_i \subseteq F$ ($i = 1, \ldots, |V|$) is the set of frequencies allocated to cell u_i, and $A_i \cap A_j = \varnothing$ if $d(u_i, u_j) < d$), find a new allocation $\tilde{A} = \{\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_{|V|}\}$ ($\tilde{A}_i \subseteq F$ ($i = 1, \ldots, |V|$), and $\tilde{A}_i \cap \tilde{A}_j = \varnothing$ if $d(u_i, u_j) < d$), such that $C(\tilde{B}, \tilde{A})$ is minimal, and A, $\tilde{A}$ are as close as possible.

The solutions to the two problems defined in this section, namely AFA and TRFA, will be the subject in sections 3 and 4, respectively.

3. AGGREGATE FREQUENCY ALLOCATION

The AFA problem is *NP*-complete [17], therefore, the application of an optimal algorithm in real-time is (usually) impractical. Nevertheless, as AFA is effective in handling a certain traffic condition (e.g., that encountered in a certain time-zone in the day), a computationally efficient solution may play an important role in the overall spectrum re-allocation scheme. The exact role will be described in section 4. In this section we discuss about possible low complexity solutions to the problem. In sub-section 3.1 we describe a reduced complexity approximate solution, which may lead to a solution that is applicable in real-time. Based on the ideas of the approximate algorithm we provide in sub-section 3.2 the rational of a class of polynomial complexity heuristics.

3.1. Approximate solution

A "greedy" approximate solution to AFA is discussed in this section. In general, greedy algorithms provide solutions by making a sequence of choices. At each phase the decision that seems best at that instant is taken. The decision, on which the greedy algorithm may be based, is to allocate each carrier frequency to a subset of V (the set of cells) that has the following properties. First, comprises cells that may use the same frequency (i.e., the cells in the set respect the frequency reuse constraints). Second, at that particular step, the allocation of the frequency to the subset in question yields the largest reduction in the blocking probability. In each cell an increase in the number of frequencies, from k to $k+1$, yields a reduction in the blocking probability that is provided by the function $r(b,k) = B(b,k \cdot N) - B(b,(k+1)N)$ [17]. The problem of determining such a subset of V may be reduced to the following *0-1 linear programming* problem [14].

$$\textit{Problem A}\text{: maximise } \sum_{i=1}^{|V|} w_i \cdot x_i \cdot r(b_i, k_i) \text{ subject to } x_i + \sum_{k \in n(i)} x_k \le 1 \; (i = 1,\ldots,|V|).$$

The decision variables x_i ($i = 1,\ldots,|V|$) will be equal to 1 for those cells u_i that belong to the set that yields the maximal reduction in the blocking probability. For the other cells, the respective variable x_i will be equal to 0. The load in the network and the number of frequencies so far allocated to cell-u_i are denoted by b_i and k_i, respectively. By $n(i)$ we denote the set of cells neighbouring to cell-u_i. The constraint of our problem ensures that no cell within a certain set will be closer than the frequency reuse distance which is taken equal to 3 edges. The corresponding algorithm for solving *problem A* is the following.

Aggregate Frequency Allocation (AFA)

Step 1: Initialisation. Formulate *problem A*.

Step 2: If $F = \varnothing$, i.e., there are no more frequencies to be allocated to the cells in V go to *step 4*, otherwise, solve *problem A*.

Step 3: Let f be the first frequency in F. Remove f from F. Allocate f to the cells in V having $x_i = 1$ in the solution of *problem A*; increase the value of the variable k_i by one, that indicates the number of frequencies that are allocated in cell-u_i ; go to *step 2*.

Step 4: End.

In this sub-section we presented an approximate algorithm to AFA. Nevertheless, we still have not devised an algorithm that is applicable in real-time due to the presence of step 2, which relies on the solution of a 0-1 linear programming problems (namely, *problem A*). The complexity induced by this step should be avoided in order to obtain a real-time algorithm. This is the subject in the next sub-section.

3.2. Polynomial complexity solution

In this sub-section we discuss about the acquisition of a polynomial complexity solution from the approximate solution presented above. The approximate algorithm for AFA (in the previous sub-section) uses *problem A* so as to determine the most appropriate set of cells that should acquire the frequency that is being allocated at a given phase of the algorithm (see figure 1(a)). In order to avoid the on-line (i.e., during the algorithm execution) computation of such sets, an initial partition of the cells of the system may be used as a basis. This partition should be obtained prior to the algorithm application. In other words, an off-line step may be introduced, in which the cell structure is pre-processed, in order to obtain a colouring or covering of the corresponding graph (see off-line segment in figure 1(b)). An algorithm that relies on such an a-priori obtained covering or colouring scheme has been considered in [18] for hexagonal and regular structures. The acquisition and processing of such a partition is also addressed in [19,20].

4. TOTAL RECONFIGURATION OF FREQUENCY ALLOCATION

4.1. General solution description

The solution to TRFA (*problem 2*) may be provided through a two phase process (see figure 2). In the first phase, the aim is to minimise the overall blocking probability in the system. Therefore, a new allocation of frequencies to cells may be found, based on the algorithms of the previous section. In other words, the first phase of the solution to

TRFA is considered as an instance of AFA. Hence, any efficient algorithm for the AFA may be applied in the first phase of the TRFA problem. Lets assume that the outcome of this phase is a new allocation of frequencies to cells $\tilde{A}_{init}$ (that minimises the overall blocking probability). However, the AFA problem does not take into account the already established allocation. In this respect, the new allocation may be markedly different, a fact that will cause significant overhead until the network adapts to the traffic variation. Hence, the second phase of the solution of TRFA should be targeted to the "harmonisation" of the new allocation with the previous one. The outcome of this phase will be the pursued allocation $\tilde{A}$. This final allocation should possess the following properties. First, it should minimise the overall blocking probability, which means that $\tilde{A}_{init}$ and $\tilde{A}$ should, in this sense, be equivalent. The manner in which the equivalence is guaranteed is explained in the next section. Second, it should be obtained by minimally impacting the already established allocation A. In the next sub-section we provide an optimal formulation of the harmonisation phase of the TRFA problem that meets these requirements.

4.2. Optimal formulation of the harmonisation phase of the TRFA problem

The harmonisation phase of the TRFA scheme may be accomplished through the following *0-1 quadratic programming problem*.

Problem B (Harmonisation phase of TRFA):

$$\textit{minimise} \quad \sum_{i=1}^{|V|}\sum_{j=1}^{|F|}(x_{ij}-\tilde{x}_{ij})^2 \tag{1}$$

$$\textit{s. t.} \quad \tilde{x}_{ij}+\sum_{k\in n(i)}\tilde{x}_{kj}\le 1, \qquad i=1,2,\ldots,|V|,\ j=1,2,\ldots,|F| \tag{2}$$

$$\sum_{j=1}^{|F|}\tilde{x}_{ij}=\left|\tilde{A}_i\right|, \qquad i=1,2,\ldots,|V| \tag{3}$$

The variables x_{ij} correspond to the already established allocation A, therefore are known. Unknown are the variables $\tilde{x}_{ij}$ which correspond to the new allocation $\tilde{A}$. The values of x_{ij} and $\tilde{x}_{ij}$ are equal to 1 (0) if frequency- j is (is not) allocated to cell- u_i. The set of constraints (2) guarantees that the frequency reuse distance constraints will be preserved. The set of constraints (3) guarantees that allocations $\tilde{A}$ and $\tilde{A}_{init}$

are equivalent, in the sense that they both minimise the overall blocking probability. More specifically, they guarantee that each cell will be allocated $|\tilde{A}_i|$ frequencies after the second phase of TRFA. This value is indicated by the first phase of TRFA, i.e., the application of the AFA scheme, and is also reflected on allocation $\tilde{A}_{init}$. Therefore, each cell possesses the same number of frequencies in allocations $\tilde{A}_{init}$ and $\tilde{A}$. *Problem B* has at least one feasible solution, namely, allocation $\tilde{A}_{init}$ obtained by the first phase of TRFA.

Unfortunately, the optimal formulation of the harmonisation phase of TRFA is *NP*-complete [15]. Hence, we describe in the following sub-section a computationally efficient heuristic solution.

4.3. Heuristic solution to the harmonisation phase of the TRFA problem

The proposed solution consists of two sub-phases (figure 3). In the first an aggregate (network-wide) harmonisation among allocations $\tilde{A}_{init}$ and A is attempted. The outcome is allocation $\tilde{A}_{mid}$, which is furthered filtered (on a cell-level basis) in the second sub-phase, so as to obtain the final allocation $\tilde{A}$.

4.3.1. First Aggregate (Network-Wide) Harmonisation Sub-Phase

Allocation $\tilde{A}_{init}$ may also be described as a collection of sets $\left\{\tilde{F}_1^{init}, \tilde{F}_2^{init}, \ldots, \tilde{F}_{|F|}^{init}\right\}$, where $\tilde{F}_i^{init}$ comprises the cells that have been allocated frequency f_i ($f_i \in F$). Of course, a condition satisfied by the cells in each set is that they are allowed to use the same frequency at the same time, i.e., the frequency reuse distance constraints are satisfied by any pair of cells $u, v \in \tilde{F}_i^{init}$ ($\forall f_i \in F$). The aim in the first aggregate (network-wide) harmonisation sub-phase is to examine whether changing the frequency allocated to each set $\tilde{F}_i^{init}$ ($\forall f_i \in F$), will result in an allocation $\tilde{A}_{mid}$ that is "closer" to A (and equivalent to $\tilde{A}_{init}$, in terms of the number of frequencies per cell). Closeness refers to the identities of the frequencies used by the cells in allocations A and $\tilde{A}_{mid}$. Preserving the frequency reuse constraints implies that every frequency f_j should be allocated (in $\tilde{A}_{mid}$) to exactly one set $\tilde{F}_i^{init}$ (as these derived in $\tilde{A}_{init}$).

The tasks in the first aggregate (network-wide) harmonisation sub-phase may be summarised in the following. First, we evaluate the impact (gain) associated with each possible change of the frequency allocated to the cells of $\tilde{F}_i^{init}$ ($\forall \tilde{F}_i^{init} \in \tilde{A}_{init}$). For example, assuming that we examine whether we should allocate frequency f_j instead of f_i to the cells of $\tilde{F}_i^{init}$, the gain should express the number of cells of $\tilde{F}_i^{init}$ that were using frequency f_j in allocation A, and hence, will regain (or maintain) a frequency previously used. Based on this computation, the second task in this sub-phase, is to formulate and solve an instance of the *bipartite weighted matching* problem [14] (which is solvable by polynomial complexity methods). A general bipartite graph for the problem may be denoted as $G_F = (F, F, E_F)$ and depicted as in figure 4. We remind that F is the set of frequencies in the system. The weights w_{ij} of each edge $(f_i, f_j) \in E_F$ represent the number of cells (in $\tilde{F}_i^{init}$) that will regain (or maintain) in allocation $\tilde{A}_{mid}$ the frequency they used in allocation A (namely, f_j), provided that (as an outcome of the harmonisation) frequency f_j will be allocated to the cells in $\tilde{F}_i^{init}$ (in $\tilde{A}_{mid}$), instead of f_i. The procedure for updating the weights w_{ij} is explained in the next paragraph. The output of the weighted matching problem is, for each set $\tilde{F}_i^{init}$, a more "preferable" frequency f_j (in the sense that f_j was used extensively in A by the cells in $\tilde{F}_i^{init}$). Furthermore, there can not be two (or more) different sets $\tilde{F}_i^{init}$, $\tilde{F}_j^{init}$ of $\tilde{A}_{init}$ that are allocated the same frequency in $\tilde{A}_{mid}$. This essential characteristic (of the output of the weighted matching problem) guarantees the preservation of the frequency reuse distance constraints.

In the sequel, we explain the procedure for updating the weights w_{ij}. Allocation $\tilde{A}_{init}$ may be brought closer to A if each $u \in \tilde{F}_i^{init}$ is allocated, instead of frequency f_i, any frequency $f_j \in A_u$, where A_u is the set of frequencies allocated to cell u in allocation A, as already defined in section 2. Hence, the value of w_{ij} denotes the number of cells $u \in \tilde{F}_i^{init}$, that had been allocated frequency f_j in allocation A, i.e., had $f_j \in A_u$. Hence, if the cells in $\tilde{F}_i^{init}$, are allocated frequency f_j, instead of f_i, w_{ij}

cells will regain (or maintain) a frequency that they were using in the previous allocation.

The objective of the problem is to find a matching with the largest possible sum of weights. The complexity of updating the w_{ij} coefficients is $O\left(|F|^2 \cdot |V|\right)$, while that of the solution of the weighted matching problem is $O\left(|F|^3\right)$. The complexity of the algorithm is low enough to justify the real-time applicability. The result is allocation $\tilde{A}_{mid}$ that has the following properties. First, is equivalent to $\tilde{A}_{init}$, in terms of the number of frequencies allocated per cell. Second, is "closer" to allocation A compared to $\tilde{A}_{init}$. Third, preserves the frequency reuse distance constraints, since, there can not be two (or more) sets of cells indicated by allocation $\tilde{A}_{init}$ that are assigned the same frequency in allocation $\tilde{A}_{mid}$.

4.3.2. Second Cell-Level Harmonisation Sub-Phase

Allocation $\tilde{A}_{mid}$, obtained in the previous harmonisation sub-phase, may be described as a collection of sets $\left\{\tilde{A}_1^{mid}, \tilde{A}_2^{mid}, \ldots, \tilde{A}_{|V|}^{mid}\right\}$, where $\tilde{A}_u^{mid}$ comprises the frequencies that have been allocated to cell u ($u \in V$), in $\tilde{A}_{mid}$. In this sub-section we describe a polynomial complexity algorithm ($O\left(|V|^2 \cdot |F|^2\right)$) that attempts to replace (if possible) from each cell u, some of the newly allocated frequencies (i.e., those that appear in $\tilde{A}_u^{mid}$ and not in A_u), by frequencies that were allocated to u in allocation A and now are not (i.e., appeared in A_u but not in $\tilde{A}_u^{mid}$). The complexity of the algorithm is such that the real-time applicability is justified. The set $C_u = A_u - \tilde{A}_u^{mid}$ comprises the frequencies that are candidate for re-allocation to cell u, in replacement of the newly allocated frequencies, which are those of the set $\tilde{A}_u^{mid} - A_u$. The maximum number of alterations that may be performed per cell is $k_u = \min\left(|A_u|, |\tilde{A}_u^{mid}|\right) - \left|\left(A_u \cap \tilde{A}_u^{mid}\right)\right|$. Before presenting in more detail the algorithm of this sub-section we provide the following definitions.

Definition 1: For each cell of the network u the corresponding *interfering* set $Int(u)$, comprises the cells that are not allowed to use the same frequency as u at the same time (e.g., the distance between u and the cells of $Int(u)$ is smaller than the minimum permissible reuse distance).

Definition 2: Given a set of cells $W = \{u_1,\ldots,u_k\}$ the set $Fr(W)$ comprises the frequencies allocated to the cells of W.

Definition 3: Given a frequency f and a set of cells W, $App(f,W)$ (appearances of f in W) denotes the number of cells in set W, to which frequency f has been allocated.

The set of frequencies C_u that are candidate for re-allocation to cell u, may be classified as follows. One subset $C_{nis}(u) \subseteq C_u$ comprises the frequencies that do not appear in the interfering set of cell u. Set $C_{nis}(u)$ may formally be described as $C_{nis}(u) = \left\{ f \middle| f \in A_u \wedge f \notin \tilde{A}_u^{mid} \wedge f \notin Fr\big(Int(u)\big) \right\}$. The frequencies in this set may directly be obtained by cell u, and replace frequencies that appear in $\tilde{A}_u^{mid}$ and not in A_u. The procedure for selecting the frequencies to be released will be explained in the sequel. The set of frequencies that belong to C_u and not in $C_{nis}(u)$ are used by one or more cells of $Int(u)$. For complexity reasons we will focus on the set of frequencies $C_{ois}(u) \subseteq C_u$ that are used once in the interfering set. This set may be formally described as $C_{ois}(u) = \left\{ f \middle| f \in A_u \wedge f \notin \tilde{A}_u^{mid} \wedge App\big(f, Int(u)\big) = 1 \right\}$. The frequencies in this set are indirectly obtainable by u, i.e., u may obtain one of them if the cell of $Int(u)$ that is in possession replaces it. In this respect, every cell $v \in Int(u)$ may have for every newly allocated frequency f ($f \in \tilde{A}_v^{mid}$, $f \notin A_v$), a set R_{vf} that comprises the cells $u \in Int(v)$ that may obtain their "old" frequency f ($f \in A_u$, $f \notin \tilde{A}_u^{mid}$), if v manages (if possible) to replace it with one of its "old" frequencies g, such that $g \in A_v$, $g \notin \tilde{A}_v^{mid}$. Based on the discussion so far we may formally describe the algorithm for the second, cell-level, harmonisation sub-phase on the TRFA formulation.

The general idea of the algorithm is depicted in figure 5. The algorithm accepts as input the sets of frequencies A_u and $\tilde{A}_u^{mid}$ for each $u \in V$, i.e., the frequencies allocated to each cell $u \in V$ according to the allocations A and $\tilde{A}_{mid}$, and provides as output the final set of frequencies that should be allocated to u according to the allocation $\tilde{A}$.

Second Harmonisation Sub-Phase ($G(V,E)$, A, $\tilde{A}_{mid}$)

Step 0: Initialisation of the data structures. Set $\forall u \in V$: $\tilde{A}_u = \tilde{A}_u^{mid}$, $C_{ois}(u) = \varnothing$, $C_{nis}(u) = \varnothing$, and $\forall f \in F$: $R_{uf} = \varnothing$.

Step 1: Update the data structures $C_{nis}(u)$, $C_{ois}(u)$ and R_{uf}. For each $u \in V$, and for each $f \in A_u$ such that $f \notin \tilde{A}_u^{mid}$, perform the following operations: If $f \notin Fr(Int(u))$ then $C_{nis}(u) = C_{nis}(u) \cup \{f\}$, otherwise, if $f \in Fr(Int(u))$ and $App(f, Int(u)) = 1$ then $C_{ois}(u) = C_{ois}(u) \cup \{f\}$; $R_{vf} = R_{vf} \cup \{f\}$ where v is the cell of $Int(u)$ such that $f \in \tilde{A}_v^{mid}$.

Step 2: Find the cell $u \in V$ such that $k_u > 0$ and $|C_{nis}(u)| = \max\{|C_{nis}(i)| \,|\, \forall i \in V\}$, i.e., the cell u that has the largest set $C_{nis}(u)$. If $|C_{nis}(u)| = 0$ for all $u \in V$ then go to step 4.

Step 3: Add the first frequency $f_1 \in C_{nis}(u)$ to the set of frequencies allocated to u, i.e., perform the operations $\tilde{A}_u = \tilde{A}_u \cup \{f_1\}$ and $k_u = k_u - 1$. Release from u the frequency f_2 of which the corresponding set R_{uf_2} has the largest cardinality. Perform the operations $\tilde{A}_u = \tilde{A}_u - \{f_2\}$; $\forall v \in R_{uf_2}$: $C_{ois}(v) = C_{ois}(v) - \{f_2\}$, $C_{nis}(v) = C_{nis}(v) \cup \{f_2\}$; Set $R_{uf_2} = \varnothing$; go to step 2.

Step 4: End.

5. RESULTS AND DISCUSSION

In this sub-section we assess the performance of the proposed scheme. Our focus is on the following aspects. First, to demonstrate the effectiveness of the TRFA formulation in efficiently utilising the available spectrum, using as a basis for comparison a GSM-like frequency allocation scheme [1]. Second, to provide some indicative examples on the behaviour of the heuristic-TRFA (H-TRFA) formulation in adapting the frequency allocation to traffic variations. Our main aim is to handle the major traffic variation (that may be assumed to be delimited by two load vectors), and to investigate the performance of the scheme during the transition from one traffic condition to the other. The third goal of the experiments is to assess the performance of the H-TRFA scheme with respect to the optimum. The basis for this comparisons will be the number of alterations in the frequency allocation pattern that are imposed by the H-TRFA and a corresponding optimal algorithm [14]. Finally, the last goal of the experiments is to provide results obtained from a scheme that uses the TRFA algorithm for handling major traffic variations and the LCAR DCA scheme ([8], see also section 1). Three sets of experiments will be used for demonstrating all these aspects. The experiments are differentiated from the size of the networks used (and not from their focus which is as described above). Initially, small networks are used so as to obtain optimal results with a reasonable computational burden (we remind that the optimal formulation of TRFA is computationally demanding).

The networks used in our experiments are square grid (Manhattan) structures. The choice of square (Manhattan) grid structures does not simplify the computational effort. Any other topology of the same size and connectivity degree could have been chosen instead. Call arrivals in the network were approximated by a Poisson process. The call duration was assumed to be exponentially distributed. The mean call holding time is denoted by $1/\mu$. Regarding user mobility we assumed the existence of one user class. The time that a user spends in a cell follows the exponential distribution, in accordance with what is proposed in [21,22,23]. The mean cell residual time is denoted by $1/n$. Regarding the distribution of the cell residual times, more elaborate models may be found in the literature [24]. However, for the purposes of this study (to propose and demonstrate the mid-term component of a spectrum management scheme) the standard models chosen are adequate. The experimentation with more advanced traffic and mobility models will be the subject of a future version of this study. In the context of our experiments we used $1/\mu$ *=180 secs* and $1/n$ *=120 secs*.

In the first set of experiments we conduct the scenarios depicted in figure 6 on a 4x4 square grid cell structure. We choose a small network for this set of experiments, in order to compare the performance of the H-TRFA scheme with the corresponding optimal algorithm. As already stated the TRFA problem is *NP*-complete, therefore, the provision of optimal solutions for large problem instances may not be feasible.

The first scenario comprises three load vectors (denoted as, L_{c1}, L_{c2} and L_{c3}, respectively) that the network has to accommodate. According to this scenario, the traffic demand is initially heavier on the periphery of the network (illustrated as Area A in figure 6(a)). Gradually, the traffic demand shifts towards the centre of the structure (Area B in figure 6(b)). The respective loads are presented in the first three columns of Table I. The load in this table is assumed to comprise the effect of mobility, following the assumptions in [16]. Table I comprises also the load that would be taken into account in a GSM-like planning process.

According to the GSM approach (worst-case based), the minimum number of frequencies for providing acceptable performance (i.e., maintaining the overall blocking probability below a certain threshold) is 17 (assuming 16 channels per frequency). This result may be obtained by merely applying the AFA algorithm for the GSM load. On the contrary, if the TRFA scheme is applied (i.e., frequency re-allocations are possible) 15 frequencies (12% less) are required for providing an acceptable level of blocking probability. This is due to the fact that the TRFA scheme handles one load vector at a time (and reconfigures the allocation so as to adapt to each new load vector). The number of frequencies required in the system is equal to the number of frequencies required for handling the most demanding load vector. This, of course, is a simple but indicative example on the manner the TRFA scheme may be used for reducing the radio spectrum (number of frequencies) required for providing acceptable QoS to the users of the system. Naturally, this means that the frequencies in the system are much more efficiently utilised.

Table II provides the performance of the TRFA scheme, expressed in terms of the estimated overall blocking probability in the network when the system has to cope with the load vectors indicated in Table I and 15 frequencies are available. Moreover, the performance of the LCAR DCA scheme is comprised. The LCAR scheme is applied to the three load vectors in the scenario. We observe that the DCA scheme further improves the system performance (as shown in the third column of Table II). In general, the H-TRFA formulation suggests a new preferable allocation of channels to cells that is capable of facing the new traffic conditions. However, the local load fluctuations, and the fact that the overall load is unevenly distributed and not excessive for the radio spectrum available, results in the improved behaviour of the

DCA scheme. This is due to the frequent adjustments of the channel allocation pattern that are pursued and achieved by the DCA scheme.

The second scenario that we exercise on the 4x4 structure comprises again three load vectors (denoted as, L_{h1}, L_{h2} and L_{h3}, respectively). The scenario simulates a horizontal shift of the traffic demand (highway like) and is illustrated in figure 6(b). The respective loads are presented in the last three columns of Table I. According to the GSM approach (worst case based), the minimum number of frequencies for providing acceptable performance is 16 (assuming 16 channels per frequency). On the other hand, through frequency re-allocations 13 frequencies (20% less) are required for the provision of an acceptable level of blocking probability. The fifth column of Table II provides the estimated overall channel request blocking probability in the network, for the various load vectors in the system, when 13 frequencies are available and 16 channels are provided per frequency, while in the last column of the same table the performance of the DCA scheme is provided. The LCAR scheme is applied to the three load vectors in the scenario.

In the sequel, we consider the cost associated with the TRFA scheme. Cost is expressed in terms of the number of changes that are required in the frequency allocation pattern so as to adapt to the traffic variation. As stated in section 4 there are two phases when attempting a frequency allocation reconfiguration. In the first phase an instance of the AFA problem is solved. The solution provided minimises the overall blocking probability. However, the requirement of having the two allocations as close as possible is neglected, which means that massive alterations in the frequency allocation pattern may be required. Consequently, significant management effort is required before the system adapts to the new environment conditions. The second phase of TRFA is applied for harmonising the new allocation with the one currently established in the network, and in this respect, to reduce the management effort required for the adaptation. Figure 7(a) depicts the number of alterations necessary for the transition from load L_{c1} to L_{c2}, in case only the first phase of TRFA is applied (i.e., no harmonisation is performed). Figure 7(b) depicts the number of alterations after the harmonisation phase. Figure 7(b) provides also a comparison among the heuristic (i.e., first, network-wide, sub-phase, and second, cell-level, sub-phase) and the optimal versions of the harmonisation phase. Figure 7(c)-(d) is the corresponding plot for the L_{c2} to L_{c3} transition. It is important to note that these figures comprise only the “unnecessary” alterations. More specifically, each new allocation suggests that some cells should be deprived from frequencies, while others should obtain some more. Hence, some alterations in the frequency allocation pattern are inevitable. In the context of our experiments we plot only the unnecessary changes. Obviously, a significant improvement is entailed by the heuristic version of the harmonisation phase of the TRFA problem, which is not far from the performance of the optimal version.

In a similar manner, figure 8(a) depicts the number of alterations necessary for the transition from load L_{h1} to L_{h2}, in case only the first phase of TRFA is applied (i.e., no harmonisation is performed). Figure 8(b) depicts the number of alterations after the harmonisation phase, which may be compared with respect to the optimal version of the harmonisation phase. Figure 8(c)-(d) is the corresponding plot for the L_{h2} to L_{h3} transition.

Figure 9 provides the system performance during the transition from load vector L_{c1} to L_{c2} and L_{c2} to L_{c3}. It is assumed that the load changes as indicated in figure 9(a), that is in a quasi-linear manner. Figure 9(a) provides the load with respect to time in Areas A and B. As indicated by the experiment, as time lapses the traffic demand shifts from (decreases in) Area A, while it gathers (increases) in Area B. Figure 9(b) provides the blocking probability accomplished at the corresponding time instants. As an overall remark reconfiguring the frequency allocation in time instants 4 and (especially) 7 results in maintaining the overall system performance in the acceptable range. From the system operation viewpoint after time instant 6 the QoS levels in the network decrease. Applying a corrective action, namely, reconfiguring the frequency allocation controls the blocking probability, and consequently, maintains the QoS levels in the acceptable range. In a similar manner, figure 10 provides the system performance during the transition from load vector L_{h1} to L_{h2} and L_{h2} to L_{h3}. The load changes in Areas A and B as indicated in figure 10(a). Figure 10(b) provides the corresponding blocking probability in the corresponding time instants. In this case the reconfigurations of the frequency allocation at instants 6 and 10 enable the preservation of the overall system performance in the acceptable range.

In the second set of experiments we use 7x7 square grid network and the traffic scenario of figure 11. Similarly to the scenarios applied on the 4x4 grid network, we assume the existence of two load vectors (denoted as, Load 1 and Load 2) that the network has to accommodate. The traffic demand is initially heavier on the periphery of the network (Area A in figure 11). Gradually it shifts towards the centre of the structure, i.e., the traffic demand is directed to Area C in figure 11 through Area B. Table III depicts the pertinent loads for each area of the network.

Similarly to our observations on the previous set of experiments (4x4 grid), the employment of TRFA (frequency allocation reconfigurations) yields considerable savings on the number of frequencies required for acceptable blocking. These would not be possible if a worst-case based frequency planning approach was followed. Table IV provides the estimated overall blocking probability in the network for the various load vectors in the system when 36 frequencies are available in the system and 16 channels are available per frequency. Again, we obtain promising results from the combined application of the DCA scheme, which is inherently more adaptive to the load fluctuations in the load range considered and for the radio spectrum that is available, and hence, succeeds in achieving better results.

Figure 12(a) depicts the number of alterations required for the transition from Load 1 to Load 2 in case the harmonisation phase is not applied, while figure 12(b) depicts the corresponding number in case the harmonisation phase is applied. Due to computational complexity reasons obtaining an optimal solution was not feasible. As in the first set of experiments (4x4 grid), the heuristic version of the harmonisation phase appears effective in reducing the required number of alterations in the frequency allocation pattern. Moreover, the performance of the heuristic scheme is close to that of the optimal one.

Figure 13 provides the system performance during the transition from load vector Load 1 to Load 2. The load changes in Areas A, B and C as indicated in figure 16(a). As time passes the traffic demand shifts from (decreases in) Area A, and moves towards Area C through Area B. Figure 16(b) provides the corresponding blocking probability in the same time windows. The reconfiguration that occurs at time instant 6 results in maintaining the overall system performance in the acceptable range. The results from this larger set of experiments are in-line and justify those obtained in the previous, smaller scale, experiments.

6. CONCLUSIONS

In this paper, we addressed the Total Reconfiguration of Frequency Allocation (TRFA) problem: “Given a set of frequencies, the cell structure, the frequency reuse distance constraints, an already established allocation of frequencies to cells, and the new traffic load conditions, find the new best allocation of frequencies to cells”. The quality criterion was the minimisation of the blocking probability in the system and the reconfiguration of the frequency allocation by performing the minimum number of alterations on the already established allocation. Due to the *NP*-complete complexity of the optimal formulation of this problem, the construction of a polynomial complexity heuristic algorithm was addressed. The proposed heuristic was analysed and assessed with respect to an optimal algorithm, in test cases where such a comparative study was possible. The proposed algorithm, solution to TRFA, is seen as a key component in a radio spectrum management scheme that performs mid-term reconfigurations, i.e., adapts the frequency allocation to traffic variations that occur in various time-zones in the day. As issues for future work we have the (more tight) integration of the TRFA scheme with the other components of the radio spectrum management scheme (e.g., the LCAR DCA scheme), the complete presentation of the radio spectrum management scheme, the realisation of a more extended set of experiments, and the experimentation with alternate traffic and mobility models.

7. References

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