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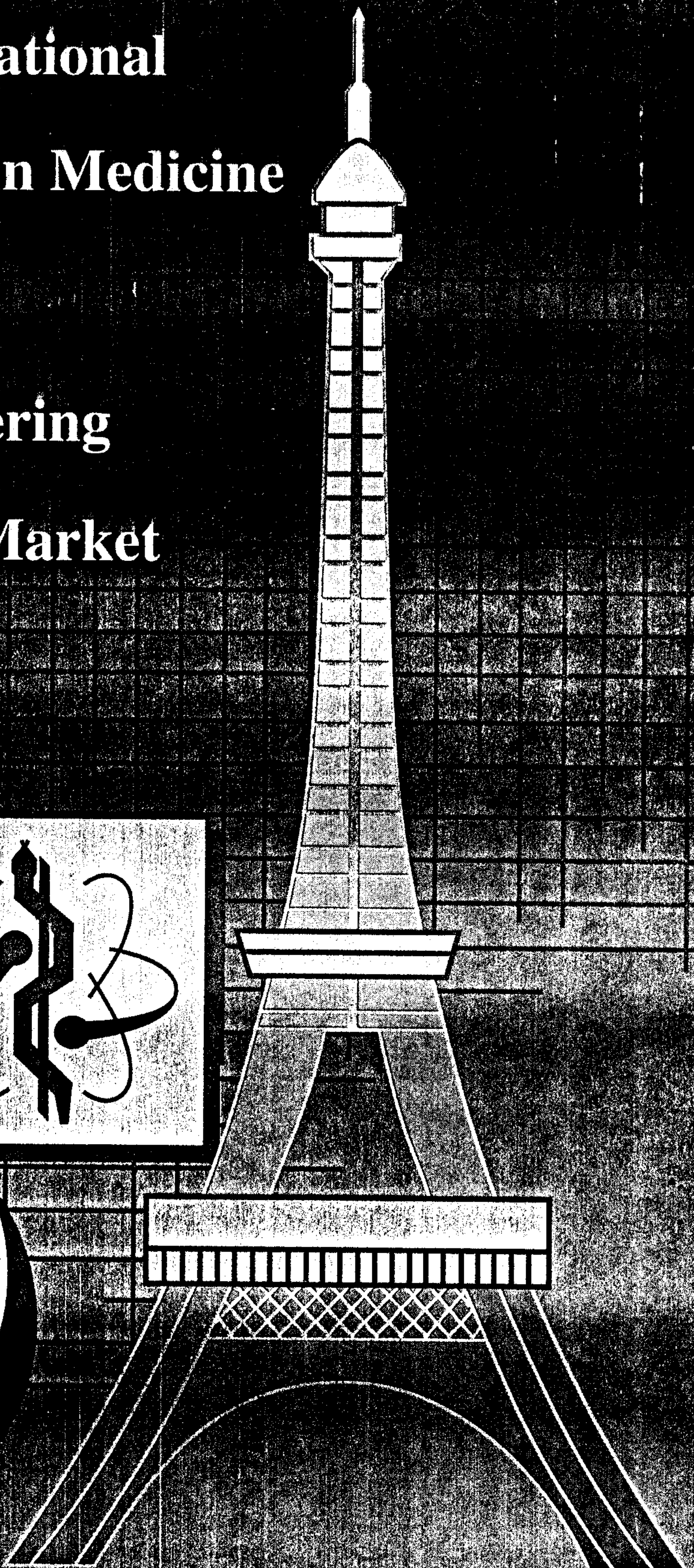

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Wigner Distribution of the Nuclear Magnetic Resonance Signal

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Abstract-Effective use of the Wigner Distribution in the signals encountered in Nuclear Magnetic Resonance is shown to reveal more information than the conventional procedures do.

INTRODUCTION

The use of the Wigner Distribution (WD) for the processing of the NMR spectroscopic signal has been proposed in [1]. The main advantages of its use may be summarised in

1) Time-frequency representation,
2) Insensitivity to phase errors.

The Wigner Transform can be considered as an extension of the Fourier Transform (FT) of a signal. It belongs to the so-called Time-Frequency distributions. In other words it gives a time-dependant frequency distribution of the signal. For this reason it has been widely used in cases where the signals encountered are nonstationary [2].

The WD of a function f(t) is defined by [3]

$$W_f(t,\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau} f(t+\tau/2)\, f^*(t-\tau/2)\; d\tau$$

and for a sequence f[n] by

$$W_f(n,\theta) = 2\sum_{k=-\infty}^{\infty} e^{-jk\theta} f[n+k]\, f^*[n-k]$$

APPLICATION IN THE MRS SIGNAL

It can be easily proved that the WD of a sum of sinusoids, as is the signal in the case of Magnetic Resonance Spectroscopy, is composed of peaks located at the frequencies of these sinusoids with the amplitudes squared and of "cross-terms" located, for each pair of sinusoids, midway on the frequency axis. These cross-terms oscillate at a frequency given by the difference of the frequencies of the sinusoids.

From this statement it is quite clear that the cross-terms will be highly undesirable in the MRS signal since the midway of two given frequencies will most likely contain useful spectral information. Thus, the cross-term effect must be eliminated for the WD to be of some validity in MR.

Conventionally, this can be done by introducing a window function, resulting to the so-called Pseudo WD. However, this approach fails to take into consideration the intristic characteristics of the MR signal. We propose here an algorithm that copes with the problem by taking advantage of the properties of the MR signal.

THE METHOD

It follows from the theoretical analysis that the cross-term due to two frequencies w_1 and w_2 will appear in frequency $w_{ct}=w_1+[(w_2-w_1)/2]$ and will oscillate at a frequency $w_d=w_2-w_1$ $(w_1<w_2)$. If at w_{ct} exists MR information, then in addition to the cross-term there will exist a decaying exponential, i.e. an unmodulated FID.

The FT of this combination will consist of a (useful) part in the baseband and a (distortion) part (an impulse function) at frequency w_d. Additional cross-terms, if any, will appear at multiples of that frequency.

Consequently, by issuing a low-pass filter one can eliminate the undesirable cross-terms and be left with the exponentially decaying term. The filtering procedure may be utilised for one or more specific distorted terms of interest or for the whole distribution in a parallel scheme.

SIMULATION RESULTS

A data sequence consisting of three decaying complex exponentials at (normalised) frequencies $w_1=-.16$, $w_2=0$ and $w_3=.16$ was created. The decay constant and the amplitude of each sinusoid were chosen to be equal. In fig.1 the WD of the signal is shown. Observe the distorted middle exponential due to the sinusoidal cross-term, which oscillate at .32, as opposed to the other two exponentials. Note also the cross-terms at frequencies $\pm.08$. The WD term at frequency w_2 is shown in fig 2a, whereas its spectrum in fig.3. Note that the spectrum is symmetrical since the WD is real. We filtered this term using a first order Butterworth IIR filter, with a cut-off frequency of .22. The output of the filter is shown in fig.2b. The decaying term at frequency w_2 has been revealed.

CONCLUSION

It has been shown that by taking into consideration the special characteristics of the NMR signal and by issuing simple DSP techniques (as is filtering) effective use of the WD may be achieved. The WD of the NMR signal may offer a more wide perspective than the conventional FT methods.

The analysis can be directly expanded to two and three dimensions, implying that the Wigner Transform may be utilised for MR Image reconstruction. In this case the ultimate output will be a set of images, each one of which will correspond to a different point in time. Using the proper time sequences one could be able to exctract more information from the MRI signal in the negligible cost of the additional post-processing required.

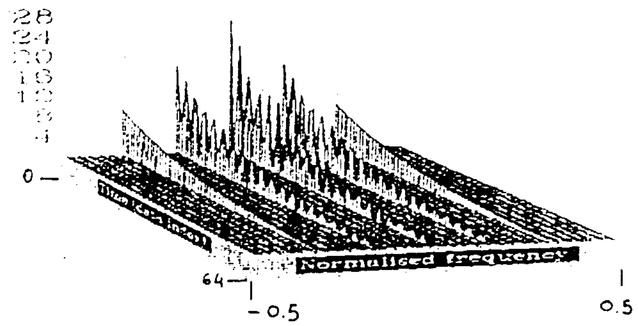

Fig.1. The Wigner Distribution of the test signal

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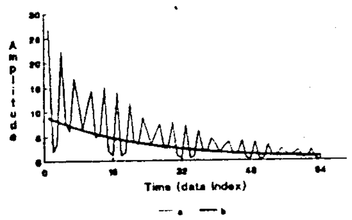

Fig. 2. The cross-term at w_2 (a) before and (b) after filtering.

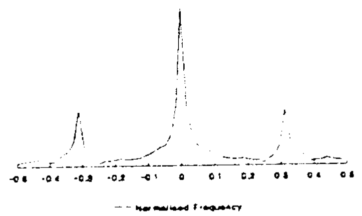

Fig.3. The spectrum of the WD term at frequency w_2.