Book of Abstracts

Volume 2

SMRM

Society of Magnetic Resonance in Medicine

Tenth Annual Scientific Meeting and Exhibition
August 10-16, 1991
San Francisco, California, USA

ISSN 0891-7612
Simultaneous proton density and $T_2$ weighted image reconstruction

P. Angelidis, K. Vassiliadis, G. Sergiadis
ARISTOTLE UNIVERSITY OF THESALONIKI
SCHOOL OF ELECTRICAL ENGINEERING, DEPT. OF TELECOMMUNICATIONS, GR. 54006, GREECE

Introduction

The MRI signal, in the presence of a linear gradient, due to $T_2$ is the inverse Laplace and not the inverse Fourier Transform (FT) of the projection. Thus, computing the Laplace Transform of the signal one can extract both the spatial and the $T_2$ information. Therefore one can obtain proton density and $T_2$ weighted images using a unique set of acquired data, that is from a single pulse sequence.

The Laplace Transform of a discrete sequence is the Z Transform ($ZT$). Implementation of the ZT instead of the DFT (Discrete Fourier Transform) is expected to improve the resolution of the frequency encoded images.

Analysis

The ZT $X(z)$ of a finite length sequence $x[n]$ non-zero for $m_{	ext{max}}$ is defined as the FT of the sequence $(r^{-m})x[n]$ for all values of $r$ that the FT exists, that is

$$X(z) = \sum_{n=m_{\text{max}}}^{m_{\text{max}}} x(n)z^{-n}, z=re^{jo} \quad (1)$$

It is straightforward to show that

$$ZT{x(n)} \rightarrow |z| = \text{FT}{r^{-m}x(n)} \quad (2)$$

This result shows that the DFT provides the means for the computation of samples of the ZT of a finite length sequence, taken at equally spaced points around a given circle, (1). The ZT is a generalization of the DFT in the same manner as the LT is a generalization of the FT.

The MRI signal is a sum of exponentially damped sinusoids, each at a different rate of decay, imposed by the $T_2$ value of the corresponding voxel. Every term of this sum is characterized by its frequency and its $T_2$ value, and is represented by a pole on the complex $z$-plane (1). Identification of such a pole can be achieved by implementing the ZT.

Evaluating the DFT is equivalent to computing the ZT for $r=1$ (eq.(2)), where $|z|=1$ (eq.(1)). As the ZT is computed for different values of $r$, sharper resonence peaks appear when a pole is approached (fig. 1). The physical meaning of this result is that as one evaluates the ZT closer to a pole the corresponding damped sinusoid is becoming an ideal undamped sinusoid which is represented by a “delta” function in the frequency domain.

Thus, $T_2$ identification can be achieved by finding the locations where sharp resonence exist.

Discussion

Obviously, the more sharp resonence is obtained the less imaginary part amplitude (dispersion mode) the FID has. Thus, one criterion for the estimation of $T_2$ is the absence of significant imaginary part in the specific frequency. Alternatively one can test the phase of the FFT, which as well lessens when the path in the $z$-plane approaches a pole.

In figure 2 is shown the behavior of the imaginary part of a particular frequency as a function of the radius $r$ in the $z$-plane, that is as a function of the $T_2$ value. It is evident that from successive samples in the $z$-plane one can easily work out where the desired minimization of the imaginary part of the FFT appears.

Because the proposed method depends on the situation of zero phase, any phase error present must be corrected as proposed, for example, by Lai and Lauterbur (2).

Conclusion

Proton density and $T_2$ weighted images should no longer be considered as two completely independent imaging techniques. A single pulse sequence is sufficient to extract both images if ZT is implemented.

The proposed method not only offers the opportunity to obtain both the spatial and the $T_2$ proton distribution of the image volume, but also to remove the broadening effect of $T_2$ on the reconstructed image resulting to higher image resolution.

The required additional post processing becomes insignificant today, given the rapid evolution of computers.

References


![Figure 1: ZT of a spin isochronat near (left) and far (right) from the corresponding pole.](image1.jpg)

![Figure 2: The imaginary part in the vicinity of a pole.](image2.jpg)