VERY FAST APPROXIMATE RECONSTRUCTION OF MR IMAGES

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Abstract-The ultra fast Fourier transform provides the means for a very fast computation of an MR image, because it is implemented using only additions and no multiplications at all. It achieves this by approximating the complex exponential functions involved in the Fourier transform sum with computationally simpler periodic functions. This approximation introduces erroneous spectrum peaks of small magnitude values. We examine the performance of this transform in some typical MRI signals. The results show that this transform can very quickly provide an MR image. It is proposed to be used as a replacement of the classically used FFT whenever a fast general overview of an image is required.

INTRODUCTION

The last years a number of fast methods have been proposed for MRI reconstructions. These can be classified as single-shot or multiple-shot techniques. The first group contains techniques such as echo planar imaging and its variations (MBEST being the most popular among them), which produce the necessary data to reconstruct the image within periods of tenths of ms [1]. The second group contains low flip angle techniques, such as Snapshot FLASH, GRASS and various SSFP techniques, which produce the necessary data to reconstruct the image within periods of hundreds of ms [2]. As a result of these short acquisition times, the reconstruction time has become an important fraction of the total time necessary to produce an image.

Furthermore, often it is very helpful for the operator to have a general overview of the final image in order to reduce the duration of clinical MR investigations, for example by modifying some of the imaging parameters. To achieve this kind of on-the-fly information, a very fast reconstruction algorithm must be available.

Usually, the reconstruction algorithm consists of a series of DFTs applied initially horizontally (readout axes) and then vertically (phase encoding axes). The DFT sum of a two dimensional (2D) sequence x[n,m] consisting of NxM samples ($n,m \in Z$, $0 \le n < N, 0 \le m < M$) is defined by

$$X[k_x,k_y] = \sum_{n=0}^{N-1}\sum_{m=0}^{M-1} W_N^{nk_x} W_M^{mk_y} x[n,m]$$

where

$$W_N = e^{-j2\pi/N},\ W_M = e^{-j2\pi/M}$$

The 2D sequence $X[k_x,k_y]$ constitutes a perfect sampling sequence of the FT of the MRI signal, i.e. the MR image, as long as the Nyquist criterion is satisfied during the acquisition.

Almost always the DFT is computed using the well-known FFT algorithm, which reduces the computational load from $O(N^2M^2)$ to $O(NM\log_2(N+M))$. In order to further reduce the reconstruction time one can approximate the complex exponential function (cef) $W_N W_M$ by a more efficient, from a computational point of view, periodic function. This way the computational effort will be lessen, but in the cost of introducing artifacts in the final result. However, in cases where a very fast general overview of the image is desired, such an approach could be useful.

We examine here the performance of an algorithm which uses the computationally most efficient approximation function. This periodic in both dimensions function takes only four values, namely 1,-1,j and -j. The reduction of the computational load is achieved in the cost of the introduction of aliasing artifacts as we will soon demonstrate. The one dimensional (1D) version of the algorithm based on this four values approximation function of the cef has been successfully applied in the processing of the MRS signals (4).

ANALYSIS

If we approximate the cef with the four-value approximation function

$$g_{4,4}[k_x,k_y] = e^{j2\pi(k_x/4+k_y/4)} = e^{j\pi(k_x+k_y)/2}$$

it turns out easily that

$$G[k_x,k_y] = \frac{8}{\pi^2}\left(X[k_x,k_y] + \frac{1}{3}\left(X[-3k_x,k_y] + X[k_x,-3k_y]\right) - \frac{1}{5}\left(X[5k_x,k_y] + X[k_x,5k_y]\right) + \frac{1}{9}X[3k_x,3k_y]\cdots\right)$$

The analysis is simple and is available upon request. A similar analysis, but for the one-dimensional case can be found in [4]. The above equation shows that transforming with the function $g_{4,4}[K_x,K_y]$ results to a scaled version of the FFT of the sequence x[n,m], plus erroneous harmonics which become less important as their frequencies increase.

The original cef (dashed) and its approximate (solid) are shown in fig.1. Since $0 \le k,l < 4$, it is obvious that the function $g_{4,4}(\omega)$ takes only the four values 1,-1,j and -j. Using this approximation the multiplications when computing the DFT defining expression become trivial; they completely vanish or become a mere sign change or a real-to-imaginary swap. Due to its speed, we call this transform the Ultra Fast Fourier Transform (UFFT).

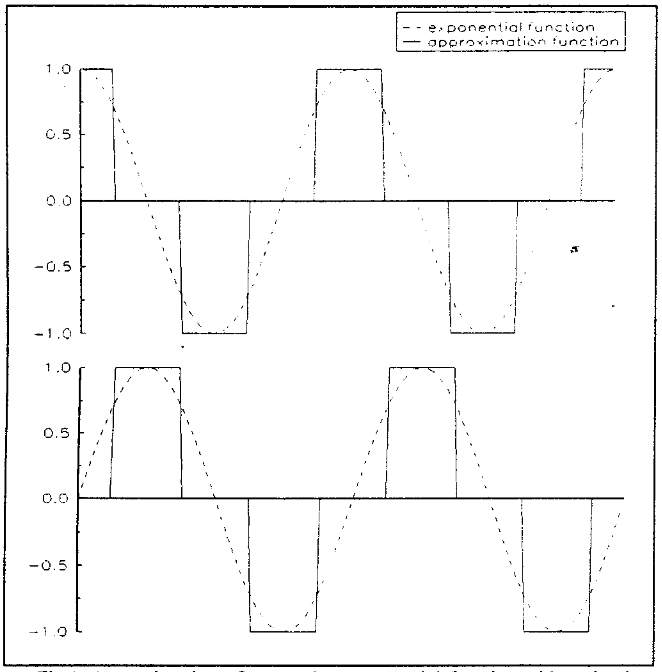

Fig.1_Approximation of a complex exponential function with a simpler function taking only four distinct values and having the same period: above real and bottom imaginary part. Only two periods of the functions are shown.

APPLICATION

We applied the algorithm to a standard spin-warp gradient-echo sequence of a 256x128 gradient-echo image of a mouse spinal cord in vitro. Data were acquired at a CISCO 4.7T machine, located at NMR Imaging Facility Centre, Queen Mary's College, London, UK. The FFT image is shown in fig.2.

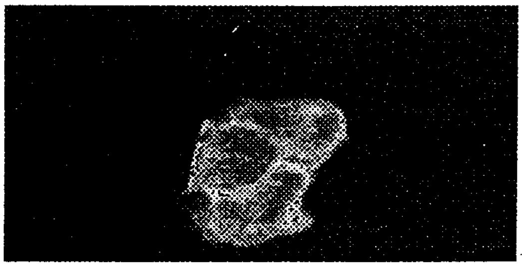

Fig.2_The spinal cord image reconstructed using a standard FFT algorithm.

In fig.3 the UFFT image is shown. Observe, that the basic structure has been revealed. In addition, erroneous harmonic terms can be seen, surrounding the real image.

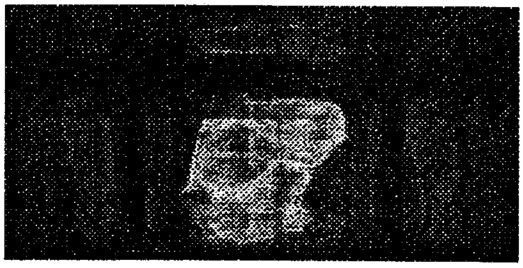

Fig.3_The image of fig. 2 reconstructed using the UFFT algorithm.

A much improved result is obtained if the FFT is applied along the readout direction and the UFFT along the phase-encoding one, as is shown in fig.4. This time the erroneous terms are less important and appear only along the vertical direction.

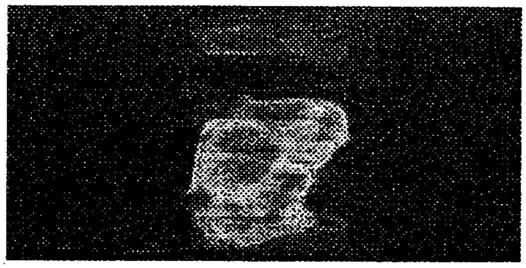

Fig.4_The image of fig. 2 reconstructed using a hybrid approach.

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