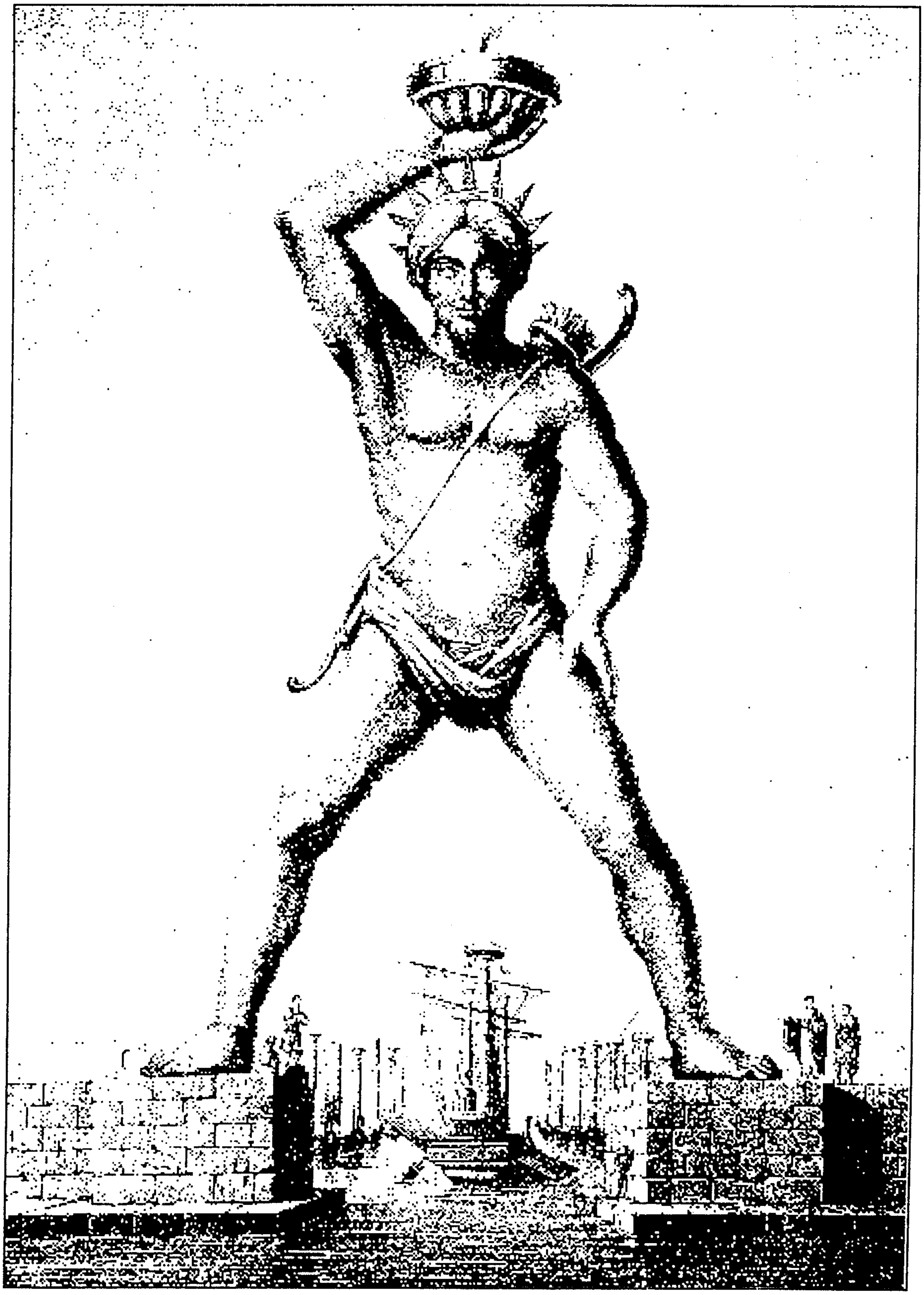
# COMCON 4 — 1993

4th International Conference on
Advances in Communication & Control
Proceedings



LE COLOSSE.

# Optimal Signal Dimension Reduction in DFT Processing Applied on Magnetic Resonance Imaging

Kostas P. Vassiliadis, Pantelis A. Angelidis, George D. Sergiadis

Aristotelian University of Thessaloniki,
School of Electrical Engineering,
Department of Telecommunications,
Thessaloniki 54006,
Macedonia, GREECE

**Abstract**

It may seem odd that one can reconstruct a 2D object by taking a single projection of that object and using 1D FT processing methods, as it is done in the Echo Planar methods in Magnetic Resonance Imaging. However, a solid interpretation can be built on the following two points. 1: The FT of a periodical function is a discrete function. In a 2D discrete function there is a specific projection angle from which the entire object can be reconstructed. 2: Any MxN 2D discrete function is shown to be equivalent to a 1D discrete function with MxN points, in both time and frequency domains. The image can therefore be reconstructed using 1D FT techniques, under the condition of periodically scanning its FT plane.

## I. Introduction

There has recently been a rapid increase of interest in Fast Imaging Methods in Magnetic Resonance Tomography [1],[2]. There are two kinds of such methods: the Echo Planar Methods and the Steady - State Free Precession Methods which have both been established in the later 1970s and applied in the clinical practise very recently. In this paper we are concerned with an interpretation of the Echo-planar methods, from a digital signal theory point of view.

In MRI (Magnetic Resonance Imaging) experiments, the FID (Free Induction Decay) in the presence of a linear gradient is the FT (Fourier Transform) of the object's projection in the direction of the gradient [1],[2]. It is clear that one needs an infinite number of projections for reconstruction. A more accurate reconstructed image is obtained when the number of projections increases. If the object is symmetrical, then there is only a finite number of different projections. For example, if the object is a 2D (2-Dimensional) circle, then all the projections have the same information and consequently the object can be represented by a single projection. Furthermore one can snow [3] that if the image is a discrete 2D function, it can then be reconstructed by a specific single projection. In a continuous distribution there will generally exist many spatial coordinates that correspond to the same point in the object's projection (view). If however the object is discrete (that is, not continuous), there exist special projection directions which allow all points of the object to be uniquely resolved. Thus, the problem of reconstruction by a single projection, becomes the problem of imposing a discrete function on a continuous spin distribution [1], that is the problem of discretizing the object.

The first attempt (in MRI) was selective excitation [4]. In this case one excites a discrete set of spins, selected from the continuous spin distribution. A specific direction then exists that allows a one to one correspondence between the frequency and the space coordinates. Due to the selective irradiation, some parts of the image do not contribute to the signal, thus resulting into a reduction of the total S/N ratio.

In the second attempt [5] the complementary situation is proposed. That is, a continuous spin density distribution is determined, while periodicity is imposed in the time domain. It is known that if a function is discrete, then its FT is a periodic function. In MRI the signal is the FT of the object's projection in the direction of the linear gradient. If this projection is discrete then the signal is periodic. The echo-planar method inverses this argument: If the signal is periodic then the reconstructed image is discrete. Therefore, there exists a specific direction from which one can view the entire object. The periodicity in time domain is achieved either by gradient reversal or by using "π" pulses, that is by recalling the signal in the form of spin or gradient echoes [6]. The disadvantages of echo-planar methods are high bandwidth requirements and, if gradient echoes are used, the difficulty of realising the high speed switching gradients and dealing with the eddy currents induced.

In conclusion, a specific orientation exists from which an entire discrete object can be seen, without overlap between its different parts. The problem of discretizing the image is solved in MRI by recalling the signal; that is by making it a periodical function. In addition the problem of reconstructing a 2D function using 1D processing methods is solved, given that any 2D discrete function can be represented as an 1D function [7].

## II. A simple description of echo-planar methods

In this section we are going to give a simple, intuitively acceptable description of echo-planar methods. The problem is how to reconstruct an object having any number of dimensions, by a single projection; that is by one view of the object. For simplicity, let us suppose that we have a 2D square and a linear array of illuminating sources. The question now becomes: which are the specific conditions that will allow the entire object to be illuminated? One can intuitively say that the object must have openings in it, which openings allow the photons to propagate through. Remembering that light propagates in a straight line, the holes must nave a structure which does not allow any lighted part of the object to shade another part (figure 1). Therefore, there exists a specific direction, from which one can illuminate every point, in every single column, using the array of light sources. Notice that our object "has holes" in 1 of its 2 dimensions. This is a general result; the N dimensional object must be discrete in at least N-1 dimensions.

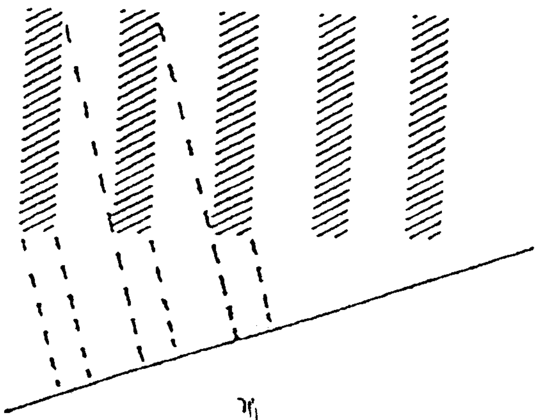


Figure 1: The structure of a 2D square which does not result into any region being shaded, when illuminated from the particular linear array of light sources.

The question now becomes: now can we apply these results in the general case where the object does not have the proper structure of openings? If we suppose that our image is a complex one, a pair of amplitude and phase values are going to characterize every point in it. Adding two images that have identical amplitudes and opposite phases in every point, will result into a "zero" image; that is an "empty hole". If we superimpose many images with identical amplitudes but with different linear phases all of them along a specific direction, the result will depend on the specific image location. In some point there will be a positive contribution from this superimposition, while in others there will be a negative one. We can say that this superimposition results into a multiplication of the original image with a function that is characterised by lack of uniformity (directivity). Thus, the final image must be build as a superimposition of many images, each with a different phase encoding, in such a way that free space will be created in the otherwise continuous image.

The FT has the property that if $f(t)$ and $F(\omega)$ are an FT pair, then the FT of $f(t-to)$ is $F(\omega)\exp(-j\omega t_0)$. Therefore N repetitions of the $f(t)$, that is $\sum_{n=0}^{N-1} f(t-nt_0)$, have an FT equal to

$$F(\omega)\left(1+e^{-j\omega t_0}+e^{-j2\omega t_0}+\ldots+e^{-j(N-1)\omega t_0}\right)=F(\omega)\sum_{n=0}^{N-1} e^{-jn\omega t_0}=F(\omega)\frac{1-e^{jN\omega t_0}}{1-e^{j\omega t_0}} \qquad (1)$$

The multiplication of this function by $F(\omega)$ results into an "observation pattern" of $F(\omega)$. This observation pattern can be viewed as a selective directivity pattern not unlike the array factor in antennas [8]. In other words, some points of $F(\omega)$ are seen as more intense than others. As N tends to infinitive, this will result into an ideal discretization of $F(\omega)$.

In MRI, we have a 2D image $F(\omega_x,\omega_y)$ and the corresponding signal $f(t_x,t_y)$. The FT shifting property says that if $f(t_x,t_y)$ transforms to $F(\omega_x,\omega_y)$, then $f(t_x-a,t_y-b)$ will transform to $F(\omega_x,\omega_y)\exp(-ja\omega_x-jb\omega_y)$. The function $\sum_{n=0}^{N-1} f(t_x-na,y)$ will transform to $F(\omega_x,\omega_y)\frac{1-e^{jNa\omega_x}}{1-e^{ja\omega_x}}$. This results into a discretization in the x-direction of the image. One can also view this process as a filtering of the image by a series of band pass filters which "cut" the image in specific locations.

Notice that since the creation of free space depends on the phase relation between successive images, one must add these images appropriately. That is the images must be added while preserving "phase coherence"; otherwise an undesirable observation pattern will appear.

To permit the reconstruction of the whole image from its samples, the free space must be of appropriate width. This width depends on the distance between the consecutive periods of the acquired signal $f(t_x,t_y)$. In fact the distance between two periods of the signal is the inverse of the distance between two successive non "empty" rows in the final image.

## III. The meaning of dimensions in the DFT

### (a) the mapping of a 2-D discrete function to a 1-D function

The 2D-FT (2-Dimensional Fourier Transform) of a 2D discrete function $x(m,n)$ with MxN points is defined [9] as :

$$X(\omega_m,\omega_n)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}x(m,n)\exp(-jm\omega_m-jn\omega_n) \tag{2}$$

which has a straightforward generalisation to more than two dimensions. The function $X(\omega_m,\omega_n)$ is periodic in both the horizontal and the vertical direction, with a period of $2\pi$.

The 2D sequence x(m,n) can be mapped to a 1D sequence y(p) with MxN terms by the relation :

$$y(p)=y(Nm+n)=x(m,n) \tag{3}$$

where $0<m<M$ and $0<n<N$. Taking the 1D FT of the sequence y(p) we have:

$$\begin{aligned}Y(\omega)=\sum_{p=0}^{MN-1}y(p)\exp(-jp\omega)&=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}y(Nm+n)\exp(-jNm\omega-jn\omega)=\\&\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}x(m,n)\exp(-jNm\omega-jn\omega)\end{aligned} \tag{4}$$

Comparing Eq.(2) and Eq.(4) it is obviou that the 1D FT of y(p) corresponds to the 2D FT of x(m,n) along the line $\omega_m=N\omega_n$ in the 2D Fourier plane. This line is at an angle $\theta=\arctan(1/N)$ to the $\omega_m$ axis. Due to the periodicity of the Fourier domain, this single line becomes a set of N parallel line segments, as is shown in figure 2. Notice that this is the familiar figure of the K-space trajectory in the first version of the Echo-Planar method used in MRI [10]. Since x(m,n) can be recovered from y(p), it is clear that there is a corresponding relation of the two sequences, in the Fourier domain. That is, one can recover the $X(\omega_m,\omega_n)$ from the $Y(\omega)$ [7]. In fact the function $Y(\omega)$ is a form of sampling of the function $X(\omega_m,\omega_n)$ along the $\omega_n$ direction.

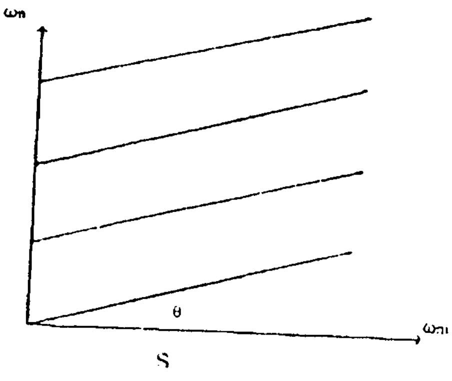



Figure 2: The N line segments of the 2D Fourier plane of x(m,n) which are derived from the 1D-FT of y(p). The angle $\theta$ is given by $\tan\theta=1/N$.

(b) projections and slices

One can easily find the specific angle from which a MxN discrete function x(m,n) can be entirely (and thus invertably) projected. The condition that must hold is that in this specific orientation, there must exist enough free space between two successive m, that is $m_i$ and $m_{i+1}$, so that the N points that in the 2D array have the index m, can be projected without overlapping. This means that, as shown in figure 3,

$\Delta m\cos\theta = N\Delta n\sin\theta$. Therefore, we are looking for the angle $\theta$ which satisfies the condition $\tan\theta = \Delta m/N\Delta n$. For $\Delta n = \Delta m$ we have $\theta$=arctantan(1/N). The function y(p) (Eq.(3)) is the projection of the function x(m,n) from this angle. In this specific direction there is an one to one correspondence between the 2D function and its 1D projection.

It is known [11] (central slice theorem) that the 1D FT of a projection is equal to the slice of the 2D FT of the original function, in the direction along which the projection has been taken. Since x(m,n) is a discrete function, there is a periodicity in $X(\omega_m,\omega_n)$. Therefore $Y(\omega)$, a particular slice of $X(\omega_m,\omega_n)$, gives a series of N equally spaced parallel line segments, which are at an angle $\theta$=arctan(1/N) to the $\omega_m$ direction (figure 2).

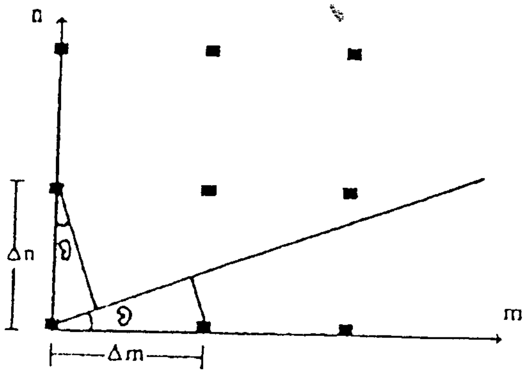



Figure 3: The evaluation of the angle $\theta$ for which there is no overlap in the projection of a 2D discrete function. The condition is that along the direction of projection, there must be enough space between two successive m, so that the N pixels corresponding to each m be rendered visible. Therefore, $\Delta m\cos\theta = N\Delta n\sin\theta$ and thus $\theta$=arctan(1/N) for $\Delta m = \Delta n$. In the figure M=N=3.

## (c) Discrete Fourier Transform

The 2D-DFT (2-Dimensional Discrete Fourier Transform) of x(m,n) is a set of MxN equally spaced independent samples of the 2D-FT of x(m,n) (figure 4a). The 2D DFT is defined [9] as:

$$X(k_m,k_n) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} x(m,n)\exp\left(-j\frac{2\pi}{M}mk_m - j\frac{2\pi}{N}nk_n\right) \qquad (5)$$

The function $X(k_m,k_n)$ is a discrete periodic function with periods M and N in the $k_m$ and $k_n$ directions respectively. The 1D-DFT of y(p) results into MxN equally spaced samples of the 1D-FT of y(p) and therefore of the 2D-FT of x(m,n). These samples are along the N parallel line segments of figure 2. There are M equally spaced samples per line as is shown in figure 4. Although these samples are different from those of $X(k_m,k_n)$, they are completely equivalent to them since they can be used to reconstruct y(p) and therefore x(m,n). These different samples have the same information content; their difference is due to a different choice of sampling direction. In fact the sampling geometry used by Y(k) is a rotated version of that used by $X(k_m,k_n)$ which was the simple rectangular sampling geometry.

Consequently, any 2-D sequence can be considered as a 1-D sequence. The DFT of the 1-D sequence corresponds to samples taken along a line in the 2-D plane of the equivalent 2-D sequence's 2D DFT. The DFT of a discrete function is periodic and consequently the lines become a series of parallel lines, a scheme familiar in the K-space trajectory of the Echo-Planar imaging method. These results can be obviously generalised to more than two dimensions. Therefore, from the DFT point of view there is no point in speaking of a multidimensional discrete sequence or of a discrete function's dimensions. Every multidimensional discrete function is in both "time" and "frequency" domains completely equivalent to a 1D function, from which 1D function it can be recovered. It is known [9] that a 1D sequence can in a similar way, be thought of as a multidimensional sequence. For DFT purposes, one can either convert a 1D-DFT to a MD-DFT or convert a MD-DFT to a 1D-DFT. As far as DFT processing is concerned, the dimension of a discrete signal is only a choice for representing and interpreting the same amount of data.

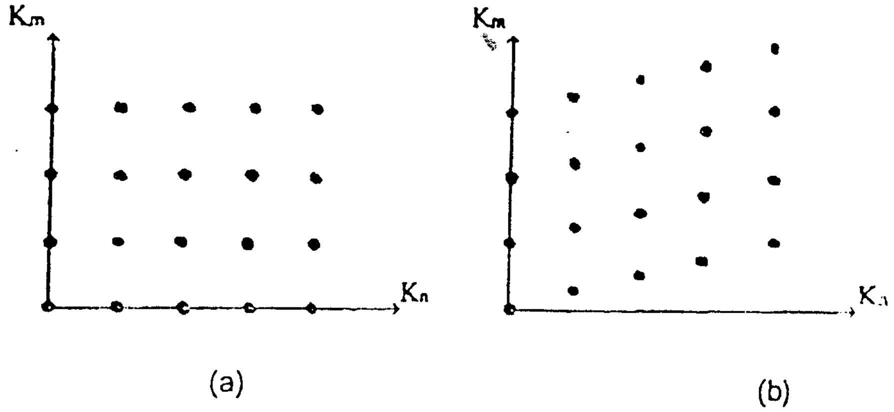



Figure 4: (a) The MxN points of the 2D FT of x(m,n) which are evaluated by the 2D DFT of x(m,n). (b) The 1D DFT of y(p) consisted of MxN equally spaced independent samples of the FT of y(p), that is of the 2D FT of x(m,n). There are N parallel line segments and each has M equally spaced points.

## IV. Periodicity and discretization in two dimensions

A periodic 2D sequence must repeat itself in two different directions at once. The 2D periodicity pattern can be defined by a pair of linearly independent vectors $P_1$ and $P_2$. These two vectors represent the displacement from any sample of the first period to the corresponding sample of two other periods. That is

$$x(m,n) = x(m + P_{11}, n + P_{21}) = x(m + P_{12}, n + P_{22}) \tag{6}$$

The vectors $P_1$ and $P_2$ are $[P_{11}\ \ P_{21}]^T$ and $[P_{12}\ \ P_{22}]^T$, where "T" denotes the transposed vector. In the simple case where the sequence repeats itself according to a rectangular pattern we have that $P_1=[P_{11}\ \ 0]^T$ and $P_2=[0\ \ P_{22}]^T$. The vectors $P_1$ and $P_2$ can be arranged to form the columns of the so called periodicity matrix P [9], which is

$$P = [P_1\ \ P_2] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{7}$$

A sample (m,n) repeats itself in (μ,ν) when

$$\begin{bmatrix} \mu \\ \nu \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \kappa \\ \lambda \end{bmatrix} \tag{8}$$

where $\kappa, \lambda$ are arbitrary integers.

In a similar manner, the sampling process of a 2D continuous function can be defined using two linearly independent vectors $S_1 = [S_{11} \ \ S_{21}]^T$ and $S_2 = [S_{21} \ \ S_{22}]^T$. The relation between the discrete (m,n) and the continuous $(t_m, t_n)$ variables is

$$\begin{bmatrix} t_m \\ t_n \end{bmatrix} = [S]\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix} \tag{9}$$

where [S] is the sampling matrix generated by using the sampling vectors as columns. In the case of a rectangular sampling we have that $S_1 = [S_{11} \ \ 0]^T$ and $S_2 = [0 \ \ S_{22}]^T$.

Periodicity in one domain, imposes a discretization in its respective FT domain and sampling in one domain results into periodicity in its respective FT domain. In fact, sampling the signal in one domain with a sampling matrix S, results to periodicity in the other domain, with a periodicity matrix P, where

$$P^T = 2\pi S^{-1} \tag{10}$$

In a similar fashion a periodic expansion of a sequence with a periodicity matrix P, imposes a discretization in its respective FT domain with the sampling matrix given by Eq.(10). That is, the choice of S (P) in one domain determines P (S) in the other domain according to Eq.(10). For instance, in the case of the rectangular sampling described by the matrix

$$\begin{bmatrix} \Delta m & 0 \\ 0 & \Delta n \end{bmatrix} \tag{11}$$

the periodicity matrix in the other domain will be

$$2\pi\begin{bmatrix} 1/\Delta m & 0 \\ 0 & 1/\Delta n \end{bmatrix} \tag{12}$$

When sampling a 2D continuous signal, one can choose any desired sampling geometry, as long as the Nyquist criterion is satisfied. The function can then be reconstructed from its samples. The sampled function is taken to be periodically extended before calculating its DFT. The DFT samples the FT of the original (aperiodical) sequence, with a sampling pattern determined by the periodicity matrix in the other domain, according to Eq.(10). In a similar fashion the DFT is a periodic function with a periodicity matrix determined by the sampling process in the other domain (again from Eq.(10)).

Finally note that these results have a straightforward generalization to more than two dimensions.

## V. Application to Echo-Planar Imaging methods

In MRI the signal is a slice in the 2D Fourier plane of the distribution or (central slice theorem) the 1D-FT of the projection of the distribution. By appropriately sampling the 2D signal domain, that is the 2D FT domain of the image which is named K-space [12],[13], the image can be reconstructed using inverse FT methods. Due to the sampling process, the reconstructed image will be a periodic function. Therefore, one must satisfy the Nyquist criterion; otherwise aliasing will occur.

Let us select the particular sampling - scanning trajectory shown in figure 5a, which is essentially the choice in the original "zig-zag" version of the Echo Planar methods [10]. The sampling matrix describing this sampling trajectory is

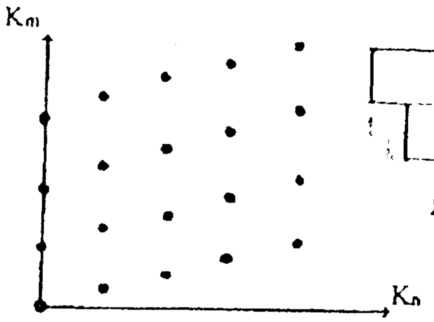
$$2\pi\begin{bmatrix} 1/M & 0 \\ 1/MN & 1/N \end{bmatrix} = \frac{2\pi}{MN}\begin{bmatrix} N & 0 \\ 1 & M \end{bmatrix} \qquad (13)$$

which in the image domain corresponds the periodicity matrix.

$$\begin{bmatrix} M & -1 \\ 0 & N \end{bmatrix} \qquad (14)$$

The image domain periodic expansion is shown in figure 5b. Notice that the vertical axis of this figure contains all the information of the 2D image, column by column. When periodicity is assumed in K space, the samples of figure 5a may be considered as lying along a single line in k-space, which is at an angle $\theta=\arctan(1/N)$ to the $k_m$ axis. Consequently there will be a "rectangular" discretization in the image domain.

In section II we mentioned the fact that the addition (superimposition) of successive images, each with a different "phase encoding", must be a "phase coherent" process. This means that the period of the signal must be an integer muitiple of the sampling period. Otherwise the periodicity of the signal will be lost and the image reconstructed will not be equivalent to a 1D function.



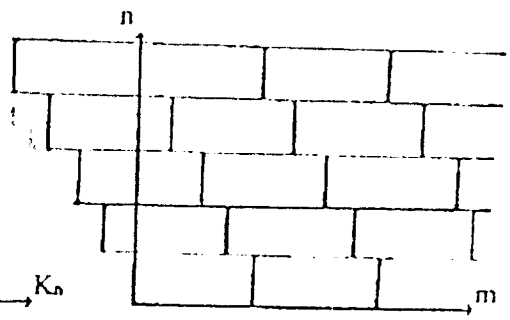




Figure 5: (a) The sampling - scanning trajectory of the original zig-zag Echo planar method. (b) The periodicity in the image domain when the previous sampling matrix of the original Echo-Planar method is used in the signal domain.

## VI. Discussion and generalization to MRI imaging methods

Since the first version of echo planar imaging, there have been many methods based on the same idea of periodicity in the time (signal) domain. Every method uses a different trajectory in the "K-space" representation. Apparently, there is nothing in these trajectories to do with the "one projection angle" $\theta=\arctan(1/N)$. For instance in the Blipped Echo-Planar Single Pulse Technique (BEST) [14] the trajectory is similar to that of the classical Spin-Wrap method. In fact the only difference is that in BEST the scanning trajectory follows a direction along odd lines which is opposite to that which it follows along even lines, whereas in Spin Wrap the scanning direction does not change. Consequently, a mere time reversal of half of the sequences acquired, will be adequate for the two scanning trajectories to become totally identical. The observation of the similarity between these two scanning trajectories naturally raises the question: Why is the reconstruction done using 1D FT methods in BEST while in the Spin Wrap method 2D FT processing is used?

In fact, the Blipped Echo-Planar version, reconstructs by applying an 1D FFT, a rotated version of the samples set. It simply rotates the sampling matrix in the image domain and it reconstructs a discrete image in which the "one projection" direction is the direction of the "m" axis. The first version of the Echo-Planar method reconstructs the image as an 1D function, column by column and receives the K-space signal in the directions $\theta=\arctan(1/N)$. Blipped Echo-Planar, receives the k-space signal line by line along the parallel lines having the direction $\theta=0$ and reconstructs the image along the parallel lines having the direction $\theta=\arctan(1/N)$. One can generally choose any trajectory in K-space which is periodic along one direction at least (for 2D images), and reconstruct the corresponding samples of the image using 1D-FFT techniques. Therefore, one can acquire the Spin-Wrap signal as an 1D signal and reconstruct MxN equally spaced samples of the image in a form similar to that of the BEST method.

In [10] we have also shown that various artifacts present in the reconstructed image in original Echo Planar method (zig-zag trajectory), are the result of the application of the 2-D FFT in the zig-zag trajectory. In this specific trajectory, as we have shown, one has to apply an 1-D FFT algorithm.

Generally, one can choose any of the sampling - scanning patterns shown in left column of figure 6, acquire the signal as a 1D signal and reconstruct the points shown in the right column of the corresponding FT domain, by using an 1D FT algorithm. These sampling trajectories are the best that may be chosen, from the point of view of the number of points needed [10].

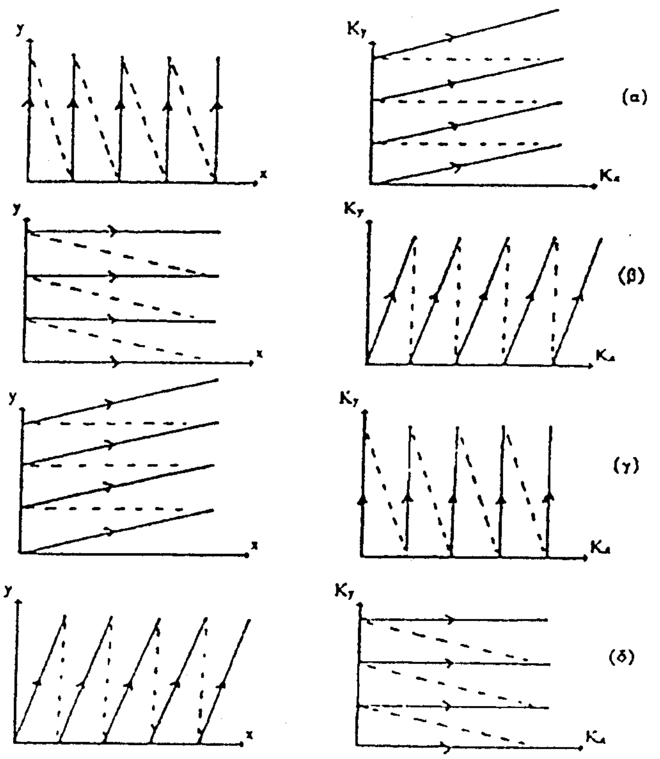



Figure 6: One can choose any of desired scanning - sampling pattern of the left column of the figure and reconstruct, by applying an 1D FT algorithm the corresponding points of the right column of the figure.

## VII. Conclusion

In this paper we proposed a general interpretation of the echo-planar methods from the point of view of digital signal processing. We have mentioned the fact that there is no meaning in speaking of dimensions of discrete signals as far as DFT processing is considered. Each discrete M-Dimensional function is completely equivalent in both "time" and "frequency" domains to a 1D sequence. We presented a simple description of how the discretization of the image domain is achieved. The reconstructed image can be thought of as a superimposition of successive images each with a different "phase encoding". An interpretation of Echo-Planar methods has been given, based on the periodicity and the sampling matrices in a 2D space. The results have been generalized to other MRI methods.

## References

[1] P. G. Morris, *Nuclear Magnetic Resonance Imaging in Medicine and Biology*, Clarendon Press - Oxford, 1986

[2] P. Mansfield, P. G. Morris, *NMR Imaging in Biomedicine, Supplement 2, Advances in Magnetic Resonance*, Academic Press, 1982

[3] R. M. Mersereau, A. V. Oppenheim, "Digital Reconstruction of Multidimensional Signals from their Projections", Proc.IEEE, Vol.62, No.10, pp.1319-1338, 1974

[4] P. Mansfield, A. A. Maudsley, J.Pnys.C: Solid State Phys. Vol.9, pp.L409-L410, 1976

[5] P. Mansfield, "Multiplanar Image Formation Using NMR Spin Echoes", J.Phys.C: Solid State Phys., Vol.10, pp.L55-L58, 1977

[6] P. Mansfield, I. L. Pykett, "Biological and Medical Imaging by NMR", J.Mag.Res. 29, pp.355-373, 1978

[7] R. M. Mersereau, D. E. Dudgeon, "The Representation of Two - Dimensional Sequences as One - Dimensional Sequences", IEEE Trans., Vol.ASSP-22, No.5, pp.320-325, 1974

[8] J. D. Kraus, *Electromagnetics*, McGraw-Hill, 1984

[9] D. E. Dudgeon, R. M. Mersereau, *Multidimensional Digital Signal Processing*, Prentice Hall, 1984

[10] K. P. Vassiliadis, P. A. Angelidis, G. D. Sergiadis, "Reconstruction of Magnetic Resonance Images Using One Dimensional Techniques", IEEE Trans. on Med. Imag., Vol.MI-12, December 1993

[11] R. A. Brooks, G. Di Chiro, "Principles of Computer Assisted Tomography (CAT) in Radiographic and Radioisotopic Imaging", Phys. Med. Biol., Vol.21, No.5, pp.689-732, 1976

[12] S. Ljunggren, "A Simple Graphical Representation of Fourier - Based Imaging Methods", J. of Magn. Res., 54, pp.338-343, 1983

[13] D. B. Twieg, "The k-trajectory Formulation of the NMR Imaging Process with Applications in Analysis and Synthesis of Imaging Methods", Med.Phys., Vol.10, No.5, pp.610-621, 1983

[14] M.Doyle et all, Lancet 2, 682, 1986