The Life of J. B. J. Fourier

PANTELIS A. ANGELIDIS

Platon S.A., TH-Sofouli 14, GR-54646 Thessaloniki, Greece

When Jean Baptiste Joseph Fourier (born 1768 in Auxerre, France; died, 1830 in Paris) was orphaned at the age of 9, he was placed by a bishop in Auxerre's military school. He had hoped to become an army engineer, but because he was not of noble birth that path was forbidden to him. He took a teaching position, which he held until the eruption of the French Revolution, when his competence at administration led to political and diplomatic assignments in Egypt, Palestine, and elsewhere.

Fourier was appointed as Prefect of Isère, France, by Napoleon in 1802. His duties included taxation, military recruiting, law enforcement, and relevant administative activities. During his service he drained 80,000 km^2 of malarial swamps and built the French section of the road to Turin, Italy.

Fourier was obsessed with heat. He used to keep his home uncomfortably hot for visitors, while being dressed heavily himself. Some traced this eccentricity back to his three years of service in Egypt. By 1807, despite official duties, Fourier had completed his theory of heat conduction, which depended on the essential idea of separating the frequency distribution into spatially sinusoidal components.

His famous work was presented in its classical "Theorie analytique de la chaleur" (1822). The basic paper was submitted to the Academy of Sciences of Paris in 1807. It was judged by the great mathematicians of his time—Joseph Louis Lagrange, Pierre Simon de Laplace, and Adrien Marie Legendre—and was rejected. However, the academy determined to encourage Fourier, and for that reason they soon after announced a grand prize for studies in propagation of heat, to be awarded in 1812.

Fourier responded with a revised paper, which he submitted in 1811. It was judged by the same panel of mathematicians and some new ones, and it finally won the prize. It was criticized, however, for lack of rigor and was not published in the *Memoires* of the academy. Although Fourier resented this treatment, he continued to work with his idea and finally published his famous paper in which he incorporated unchanged the first part of the paper that had won the prize.

Fourier derived his theory trying to solve the heat solution for a cylindrical rod whose ends are kept at 0°C and whose lateral surface is insulated so that no heat flows through it. He began with the general equation of heat propagation in a bar

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = k^2 \frac{\partial T}{\partial t} \qquad [1]$$

Changing from a bar to a rod of length l, Eq. [1] becomes Eq. [2]

$$\frac{\partial^2 T}{\partial x^2} = k^2 \frac{\partial T}{\partial t}, \ 0 < x < l \qquad [2]$$

Solving the equation he found the following general solution for the distribution of heat along the rod as a function of time:

$$T(x,t) = \sum_{v=1}^{\infty} b_v e^{-(v^2 \pi^2 k^2 l^2) t} \qquad [3]$$

Introducing the initial condition $T(x,0) = f(x)$, he deduced that in order to satisfy it, the following should be true

$$f(x) = \sum_{v=1}^{\infty} b_v \sin \frac{v \pi x}{l} \qquad [4]$$

Concepts in Magnetic Resonance, Vol. 8(5) 383–384 (1996)
 CCC 1043-7347/96/050383-02

Because he did not make any assumptions for the initial distribution of the heat $f(x)$, he concluded that the above equation should be satisfied by any arbitrary function. Apparently, this result was already used by Alexis-Claude Clairaut and Leonhard Euler for some functions, but was never generalized. In fact, Fourier never gave a complete proof of his theorem for an arbitrary function.

This was left for another great mathematician, Gustav Peter Lejeune-Dirichlet (1805–1859), who met Fourier in Paris in 1822 and was greatly influenced by him. Dirichlet showed that the Fourier series for an arbitrary function $f(x)$ converges in the interval $(-\pi,\pi)$ at each x. In Dirichlet's words: "Nothing has appeared to us more suitable than geometrical constructions, to demonstrate the truth of the new results and to render intelligible the forms which analysis employs for their expressions."