Single-Channel Demodulator and Hartley Transform in MRI

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***Abstract*—This paper presents a single-channel alternative detection system for magnetic resonance imaging and spectroscopy experiments. After the phase error identification, which is realized through a simple method, one can correct the reconstructed image and discriminate between positive and negative frequencies without any loss of the S/N ratio. Since we have only one channel, that is a real signal, we can use the Hartley instead of the Fourier transform. The former is faster and requires less storage memory.**

I. Introduction

IN MRI experiments the FID is the inverse FT (Fourier Transform) of the spatial distribution or of the projection of the spatial distribution in the direction of the readout gradient. Taking the one, two- or three-dimensional FT of the signal, depending on the imaging method, we can reconstruct the image.

Generally speaking, the image will contain both positive and negative frequencies, controlled by the gradient relative contribution. Therefore, in order to discriminate between positive and negative frequencies, we have to place our f_{ref} (the frequency by which we demodulate the signal) in the one extreme of the spectrum. Obviously this calls for expensive high frequency and wide bandwidth acquisition hardware. Alternatively, in "quadrature demodulation" [1], [2] the received signal is demodulated by a f_{ref} placed in the center of the image spectrum and on the same time by the same f_{ref} shifted by $\pi/2$. This dual channel system allows us to discriminate between positive and negative frequencies and simultaneously to improve the S/N (signal-to-noise) ratio by the factor $\sqrt{2}$. However, one needs two identical hardware channels which are difficult to realize and adjust and one must pay attention to the rate and the order of the samples taken [3]. The signal after the quadrature demodulation is considered as a complex one.

The reconstructed image is not real, that is the FT of the FID acquired either by the quadrature or by the single channel demodulator contains both a real and an imaginary part. This is due to the relaxation effects and phase errors that exist. T_2 results to a convolution of the real image with the FT of $\exp(-t/T_2)$ [4]. In this paper we are going to neglect the T_2 effect.

The phase errors are generally of two kinds [5]. The first type is a constant, frequency independent phase shift due to the difficulty of perfect phase adjustment, between the carrier and f_{ref}. This phase error mixtures both the absorption and the dispersion mode. The second kind of phase error is a linear frequency function due to hardware LPF (low-pass filtering) and the inevitable deadtime before the observation of the FID starts. This phase error can be considered as a time delay phase error. In quadrature detection, taking the amplitude of the FFT output we obtain a reconstructed image independent of those phase shifts. This is one more reason for choosing the quadrature detection.

Several papers and letters [6], [7] have proposed a single-channel demodulator by altering either the sign of the samples or the phase of the single f_{ref} before the next acquisition. In this paper we proposed a simple, i.e., one without phase or sign alternation, one channel demodulator by making use of the "hidden" symmetries of the MRI signal.

The Hartley transform (HT) is equivalent to the FT in the case of real data [8]. Therefore, in the case of MRI quadrature demodulation we cannot replace the FT by the HT, since the signal has a complex nature. However, in the case of single channel demodulator system the signal has no more a complex nature and one can use HT instead of FT. This would result in an improvement both in time and memory needed for the reconstruction of the image.

II. MRI Demodulation and Processing

A. Quadrature Detection

Generally the signal of MRI acquisition is given by

$$\operatorname{Re}\left\{\int p(w)\exp(jw_o t)\exp(jwt)\exp(j\varphi)\exp(jgw)\,dw\right\} \tag{1}$$

where w are the frequencies in the direction of the readout gradient, w_o is the Larmor frequency due to the static field B_o, φ is a constant, frequency independent phase shift, g is the factor of the linear frequency dependent phase shift and "Re" denotes the real part. After the quadrature demodulation (Fig. 1) we have two signals and

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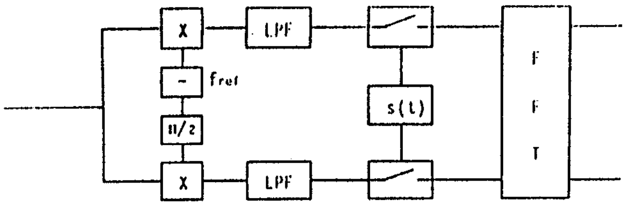

Fig. 1. Quadrature detection. The signal is demodulated by the f_{ref} and on the same time by the f_{ref} shifted by $\pi/2$. The two channels after the low-pass filtering are fed to a FFT as the real and the imaginary parts. $S(t)$ represents the sampling function.

we treat them as one complex signal, that is one channel is the real and the other is the imaginary part of the signal. After the LPF (low-pass filter) the signal is given by

$$\int p(w) \exp[jw(t+g)] \exp(j\varphi)\, dw. \qquad (2)$$

It can be easily shown from the shifting properties of the FT [9] that due to phase errors we have instead of $S(t) = \text{IFT}\{p(w)\}$, the signal (complex in nature) $S(t+g) \exp(j\varphi)$. Taking the FT of this signal we have as a result $p(w_i) \exp(jgw) \exp(j\varphi)$, that is

$$\text{Real part: } \text{Re}(w) = p(w) \cos(\varphi + gw)$$

$$\text{Imaginary part: } \text{Im}(w) = p(w) \sin(\varphi + gw).$$

In the presence of this phase shift, the reconstructed image has a complex nature, therefore in order to reconstruct the image we need to sample the whole K-space by reversing the sign of the gradients. However, the specific relations between the two parts of the complex image allow us to remove the phase error by taking the amplitude of the complex spectrum sampling only the positive K-space. The spectrum of the two channels and the way the quadrature detection discriminates between positive and negative frequencies and at the same time removes the phase errors are shown in Fig. 2.

B. Single Channel Demodulator

Demodulating the signal by a single f_{ref} (single-channel demodulator, Fig. 3) we have (after the LPF):

$$\text{Re}\left\{\int p(w) \exp[jw(t+g)] \exp(j\varphi)\, dw\right\}. \qquad (3)$$

That is the signal is $\text{Re}\{\text{IFT}[p(w) \exp(jgw) \exp(j\varphi)]\}$. It is useful to write the signal in the form

$$\int [p(w) \exp(jwt) \exp(jgw) \exp(j\varphi) + p(w) \exp(-jwt) \exp(-jgw) \exp(-j\varphi)]\, dw \qquad (4)$$

since $p(w) = p^*(w)$ due to the reality of the image (* denotes complex conjugate). In the absence of phase errors, this signal has an even symmetry and it corresponds to the even part of the image. If there is a constant phase error which equals $\pi/2$, then the signal has an odd symmetry and it corresponds to the odd part of the image. The presence of a constant phase error φ, different from $k\pi/2$

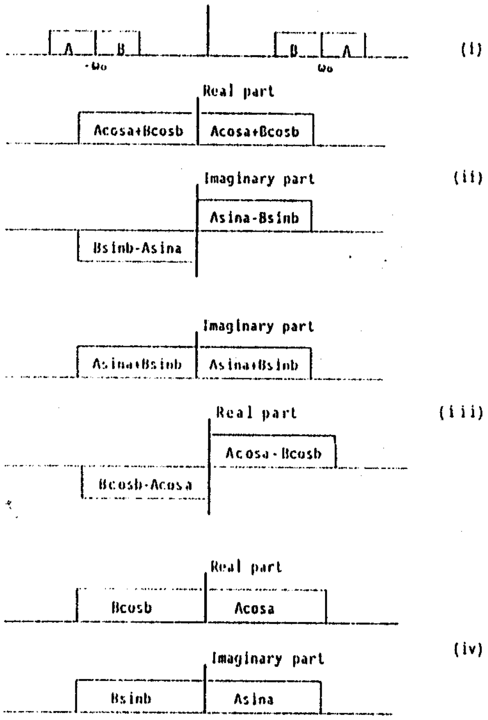

Fig. 2. The spectrum of the signals. A denotes the amplitudes of the positive frequencies, B the amplitudes of negative ones and a, b the corresponding phase error functions. (i) The spectrum before the demodulation, (ii) the spectrum of the signal demodulated by cos ω_o (real and imaginary part), and (iii) the spectrum of the signal demodulated by sin ω_o (real and imaginary part). In quadrature detection the addition of the real and imaginary parts of the two channels discriminates between positive and negative frequencies (iv). Extracting the magnitude we may remove the effect of phase errors. Having a single channel we must identify the phase error.

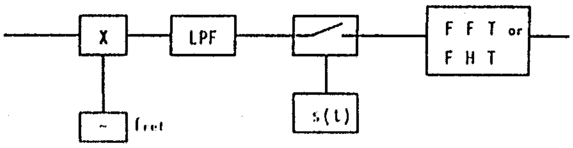

Fig. 3. The "single channel" demodulator. Notice that the FFT can be replaced by a FHT [paragraph 3(b)].

where k is an integer, has as a result the simultaneous encoding in the same channel of both the even and odd part of the image. Therefore, by decomposing the signal we obtain the information of the odd and the even part of the image. That is, due to phase error, we have in the same channel the information of the whole image. The effect of the linear frequency dependent phase error is the time shifting of the signal. After the FT we have

$$\text{Real part} = \text{Re}(w) = p(w) \cos(\varphi + gw) + p(-w) \cos(\varphi + gw) \qquad (5)$$

$$\text{Imaginary part} = \text{Im}(w) = p(w) \sin(\varphi + gw) - p(-w) \sin(\varphi + gw). \qquad (6)$$

The problem here is that generally we cannot remove the effect of the phase error and we cannot discriminate between positive and negative frequencies, since the spectrum is symmetrical due to the real nature of the signal. Thus, we have to demodulate with $w_o + \max(w_i)$, which has as a result the reduction of the S/N factor by $\sqrt{2}$ and then to correct the phase errors. Notice that in every frequency of the real (imaginary) part we have the addition (subtraction) of the positive and negative frequencies that correspond to that absolute frequency value.

C. Phase Identification and Image Correction

A solution to the above problem would be the phase identification before the acquisition, so that one could correct the reconstructed image. We are going to show that we can remove the effect of the phase error and at the same time discriminate between positive and negative frequencies using a single channel demodulator system and placing our f_{ref} in the center of the image spectrum, that is without reducing the S/N factor.

If we prepare our system so that it contains only positive frequencies relative to f_{ref} we will have as an output of the FFT

$$\text{Real part} = \text{Re}\,(w) = p(w)\cos(\varphi + gw) \quad (7)$$

$$\text{Imaginary part} = \text{Im}\,(w) = p(w)\sin(\varphi + gw). \quad (8)$$

By division we obtain the values of $\tan(\varphi + gw)$ where w is positive and we do the same for negative w. Furthermore, we can obtain the phase error for any frequency by evaluating it for only two samples. This is possible since the phase error is the sum of a constant and of a linear in frequency term.

As we have seen, after the FT of the single channel signal, we have in every spectral frequency the addition of the amplitude of both the positive and the negative corresponding image frequencies. Supposing that we have an amplitude $A(w)$ from the positive frequencies with phase error $a(w)$ and an amplitude $B(w)$ from the negative frequencies with phase error $b(w)$, then for every spectral frequency the following equations are valid:

$$\text{Real part} = \text{Re}\,(w) = A(w)\cos[a(w)] + B(w)\cos[b(w)] \quad (9)$$

$$\text{Imaginary part} = \text{Im}\,(w) = A(w)\sin[a(w)] - B(w)\sin[b(w)]. \quad (10)$$

Knowing the phase terms we can evaluate $A(w)$ and $B(w)$. We obtain the following solutions for the amplitudes:

$$A(w) = \frac{\text{Re}\,(w)\sin[b(w)] + \text{Im}\,(w)\cos[b(w)]}{\cos[a(w)]\sin[b(w)] + \sin[a(w)]\cos[b(w)]} \quad (11)$$

$$B(w) = \frac{\text{Re}\,(w)\sin[a(w)] - \text{Im}\,(w)\cos[a(w)]}{\cos[a(w)]\sin[b(w)] + \sin[a(w)]\cos[b(w)]}. \quad (12)$$

II. Hartley Transform in MRI

A. The Hartley Transform

The Hartley transform (HT), in its integral form, was introduced in [8]. It is an equivalent to the Fourier transform for real data. Let $x(t)$ denote a real function. Its Fourier transform $F(\omega)$ is given by [9]

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \quad (13a)$$

and the inverse equation is

$$x(t) = \int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\,d\omega \quad (13b)$$

where $e^{ja} = \cos(a) + j\sin(a)$.

The Hartley transform pair is defined by [10]

$$H(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\,\text{cas}\,(\omega t)\,dt$$

$$x(t) = \int_{-\infty}^{\infty} H(\omega)\,\text{cas}\,(\omega t)\,d\omega \quad (14)$$

where $\text{cas}\,(a) = \cos(a) + \sin(a)$.

The link equations of the two transforms are

$$\text{Re}\,\{F(\omega)\} = [H(\omega) + H(-\omega)]/2 \quad (15a)$$

$$\text{Im}\,\{F(\omega)\} = -[H(\omega) - H(-\omega)]/2. \quad (15b)$$

Having in mind the defining equations of the discrete Fourier transform [11]

$$F(v) = \frac{1}{N}\sum_{T=0}^{N-1} x(T)e^{-j2\pi vT/N}$$

$$x(T) = \sum_{v=0}^{N-1} F(v)e^{j2\pi vT/N} \quad (16)$$

the discrete Hartley transform (DHT) can be defined by

$$H(v) = \frac{1}{N}\sum_{T=0}^{N-1} x(T)\,\text{cas}\,(2\pi vT/N)$$

$$x(T) = \sum_{v=0}^{N-1} H(v)\,\text{cas}\,(2\pi vT). \quad (17)$$

Despite the fact that the Hartley transform was first defined in 1942, its use in practical applications became clear only in 1984, when R. N. Bracewell developed a fast algorithm for the implementation of the discrete Hartley transform known as the fast Hartley transform (FHT), which was similar in concept to the fast Fourier transform (FFT). Since it is out of the scope of this paper to give a thorough presentation of the FHT, the reader who might be interested can refer back to two excellent publications [12], [13].

Since 1984, a number of articles reporting improvements in the original algorithm have appeared in the literature. A comparative study of these may be found in [14]. It is generally agreed that FHT is faster than FFT.

B. Application of the Hartley Transform to MRI Signal Demodulation

Since for the case of real data we can acquire the real and the imaginary part of the FT as the even and the odd part of the HT, it is clear that in the case of the single channel demodulator system we can use the HT instead of the FT without any loss of information. The HT of the signal is

$$p(w)\,[\cos(\varphi + gw) - \sin(\varphi + gw)] + p(-w)\,[\cos(\varphi + gw) + \sin(\varphi + gw)] \quad \text{for } w > 0 \tag{18a}$$

$$p(w)\,[\cos(\varphi + gw) + \sin(\varphi + gw)] + p(-w)\,[\cos(\varphi + gw) - \sin(\varphi + gw)] \quad \text{for } w < 0. \tag{18b}$$

The HT is not symmetrical as far as the zero frequency is concerned because it is the sum of an even and an odd function. This lack of symmetry provides information for both positive and negative frequencies. Using (15a) and (15b) we can obtain the corresponding FT.

C. *The Hartley Transform in Higher Dimensions*

The HT and DHT can be expanded to two or more dimensions. According to the two-dimensional discrete Fourier transform (2-D-DFT) the two-dimensional discrete Hartley transform (2-D-DHT) of a real sequence $f(x, y)$ is defined by

$$H(v_x, v_y) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) \cdot \operatorname{cas}(2\pi v_x x/N + 2\pi v_y y/M). \tag{19}$$

One method of performing a 2-D-DFT is to take the 1-D-FFT of the rows one by one and then transform the columns [15]. By applying this method using FHT instead of FFT the result will be

$$T(v_x, v_y) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) \operatorname{cas}(2\pi v_x x/N) \cdot \operatorname{cas}(2\pi v_y y/M). \tag{20}$$

Observe the difference between (19) and (20). The method described above will not result in the 2-D-DHT, due to the fact that cas $(a + b) \neq$ cas (a) cas (b), whereas (in the case of 2-D-DFT) $e^{(a+b)} = e^a e^b$. An additional step is needed in this case. An elegant solution to this problem is presented in [16]. An algorithm for a 3-D-DFT using FHT may be found in [17].

The DHT can be used in all MRI methods which implement FFT techniques (e.g., Fourier zeugmatography, spin-wrap, echo-planar). Its use in medical imaging in general has already been proposed [18].

The real-type nature of the Hartley transform offers an alternative approach to frequency analysis which is closer to real world images and computer arithmetic. Its increased speed and economy of storage space are additional advantages.

V. Conclusion

This paper presents an alternative and simple approach to the detection of MRI signals. This method maintains the optimum S/N ratio using a single detection and acquisition channel. The difficulties of the quadrature detection can be overcome through this method. To be more specific, we no more need to deal with the realization and adjustment of the two identical channels and the expensive high frequency sampling required in heterodyned acquisition. Due to the fact that the demodulator system needs only one channel the FFT can be replaced by the FHT. The latter is better in terms both of time and memory requirements.

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