# Lowest Mutual Coupling Between Closely Spaced Loop Antennas

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*Abstract*—**The conditions for zeroing the mutual flux between two closely spaced coil antennas are examined. Our analysis is expanded to examine the case of multiple linear antenna coils. Array antennas are used in many areas including magnetic resonance imaging (MRI), in which we are interested. Theoretical as well as experimental results are presented and compared. Good agreement between theory and measurements has been confirmed. These results allow us to suggest a two-dimensional receiving array antenna.**

## I. Introduction

In some high frequency applications several antennas must work together to form a linear array. In such situations, compact coverage of an area is needed as well as an independence between the different receiving points. This independence is obtained if noninteractive antennas form the array, i.e., inductive and capacitive coupling between antennas must be avoided, and each antenna has its own RF receiver, i.e., the data from each of them are separately received, digitized, and stored for further processing.

A situation as the preceding one appears in magnetic resonance imaging (MRI). To obtain high signal-to-noise ratio images or to extract the same information faster, multiple receiver coils are used. A number of array antennas for MRI have appeared in the literature. Hyde *et al.* [1] have proposed and used three different types of local coil geometries for parallel image acquisition simultaneously and independently from each coil.

Roemer *et al.* [2] proposed a two-dimensional phased array consisting of 10 coils with minimized mutual coupling. Recently they presented a thorough analysis of its performance [10]. A variety of algorithms has been developed for combining the simultaneously acquired data from antenna arrays and forming images [2], [3], [10] in MRI.

Mutual coupling compensation for transmitting planar arrays in satellites has been reported by Davies [4]. In this work the compensation of the mutual coupling between the antennas is achieved through circular symmetry and phased feeding. The distant field performance of loop antenna arrays has been studied elsewhere on both horizontally oriented [5] and coaxial loops [6].

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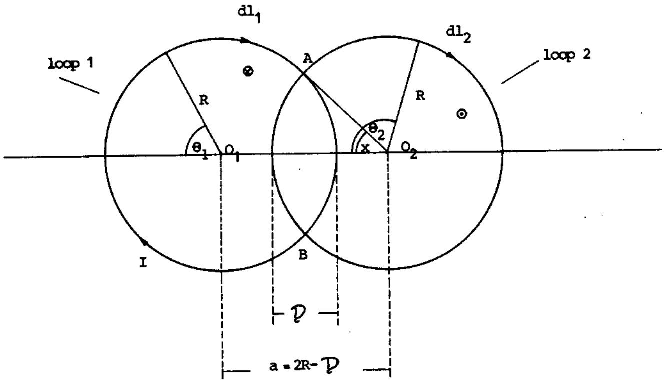


Fig. 1. The geometrical representation of the problem. The two loops are placed on the same plane. Symbols ⊙ and ⊗ show the direction of the magnetic field due to current I.

In order to achieve the magnetic independence, it is necessary to find the conditions under which the mutual flux between two coil antennas is zero. We present a theoretical solution to this problem and compare the final result to experimental measurements. Finally, the case of more than two loops is considered both in one and two dimensions.

## II. The Problem

Consider two circular loops with radius $R$ placed on the same plane sharing a common area as shown in Fig. 1. We assume a constant current ($I$) flowing through loop 1. We want to find the relative position of the two loops where the magnetic flux through loop 2 ($\Phi_2$) is zero.

It is evident that flux $\Phi_2$ is given by

$$\Phi_2 = \frac{\mu I}{4\pi} \oint_{l_1} \oint_{l_2} \frac{\overrightarrow{dl_1} \cdot \overrightarrow{dl_2}}{r}. \qquad (1)$$

The integrations are taken over circles 1 and 2, $r$ is the distance between the elementary vectors $\overrightarrow{dl_1}$ and $\overrightarrow{dl_2}$, and $\mu$ is the magnetic permeability.

The magnetic vector potential $\overrightarrow{A_1}$ at any point in space due to current $I$ is given by

$$\overrightarrow{A_1} = \frac{\mu I}{4\pi} \oint_{l_1} \frac{\overrightarrow{dl_1}}{r}. \qquad (2)$$

If we introduce angles $\vartheta_1$ and $\vartheta_2$ (Fig. 1), then the following equations for the internal product $(\overrightarrow{dl_1} \cdot \overrightarrow{dl_2})$ and

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the distance $r$ can be written

$$\overrightarrow{dl_1} \cdot \overrightarrow{dl_2} = R^2 \cos(\vartheta_2 - \vartheta_1)\, d\vartheta_1\, d\vartheta_2 \tag{3}$$

$$r = \sqrt{\alpha^2 + 4R^2 \sin^2\left(\frac{\vartheta_2 - \vartheta_1}{2}\right) + 4\alpha R \sin\left(\frac{\vartheta_2 + \vartheta_1}{2}\right) \sin\left(\frac{\vartheta_2 - \vartheta_1}{2}\right)}$$
$$= \sqrt{\alpha^2 + 2R^2 - 2R^2 \cos(\vartheta_2 - \vartheta_1) + 2\alpha R(\cos(\vartheta_2) - \cos(\vartheta_1))} \tag{4}$$

where $\alpha = 2R - \mathscr{D}$ and $\mathscr{D}$ is the distance we want to find (see Fig. 1). For negative values of $\mathscr{D}$ the two loops do not have any common area, and for $\mathscr{D} = 0$ the two loops are tangent.

From (1), (3), and (4) the final form of the magnetic flux results as a function of $\mathscr{D}$

$$\Phi_2(\mathscr{D}) = \frac{\mu I R^2}{4\pi} \int_0^{2\pi} \int_0^{2\pi} f(\vartheta_1, \vartheta_2)\, d\vartheta_1\, d\vartheta_2$$

$$f(\vartheta_1, \vartheta_2) = \frac{\cos(\vartheta_2 - \vartheta_1)}{\sqrt{\alpha^2 + 2R^2 - 2R^2 \cos(\vartheta_2 - \vartheta_1) + 2\alpha R(\cos(\vartheta_2) - \cos(\vartheta_1))}}. \tag{5}$$

The solution of the equation

$$\Phi_2(\mathscr{D}) = 0 \tag{6}$$

gives the expected optimum distance.

## III. The Solution

The analytical solution of the double integral leads to a simple integral of a function containing elliptical integrals. This was an expected problem since the magnetic vector potential of a current loop $(\overrightarrow{A_1})$ is a function containing elliptical integrals [7]. To the best of our knowledge there is no analytical solution to the above problem.

Numerical methods are better suited to the solution of the problem. Due to symmetry reasons the limits of $\vartheta_2$ can be restricted to the interval $[0, \pi]$, resulting in a factor of two. Therefore, the computational effort is halved.

The main problem to be dealt with is the determination of the function at points $A$ and $B$ (Fig. 1) where $r$, defined in (4) equals zero. Point $A$ corresponds to $\vartheta_1 = \pi - x$, $\vartheta_2 = x$, whereas $B$ to $\vartheta_1 = \pi + x$, $\vartheta_2 = 2\pi - x$ and $x = \cos^{-1}(\alpha/2R)$. One method to overcome this difficulty is to split the second integral into two parts as shown in the following [8]:

$$\Phi_2(\mathscr{D}) = \frac{\mu I R^2}{2\pi} \lim_{u \to 0} \left[ \int_0^{2\pi} \int_0^{x-u} f(\vartheta_1, \vartheta_2)\, d\vartheta_1\, d\vartheta_2 + \int_0^{2\pi} \int_{x+u}^{\pi} f(\vartheta_1, \vartheta_2)\, d\vartheta_1\, d\vartheta_2 \right].$$

Then we evaluate the following sum for different values of $\mathscr{D}$

$$2\Delta\vartheta_1 \Delta\vartheta_2 \left[ \sum_{\vartheta_1 = 0, \Delta_1}^{2\pi} \sum_{\vartheta_2 = 0, \Delta\vartheta_2}^{x-u} f(\vartheta_1, \vartheta_2) + \sum_{\vartheta_1 = 0, \Delta\vartheta_1}^{2\pi} \sum_{\vartheta_2 = x+u, \Delta\vartheta_2}^{\pi} f(\vartheta_1, \vartheta_2) \right] \tag{7}$$

decreasing the value of $u$ until a satisfying convergence is established. In other words, $u$ shows how closely the infinity at point $A$ must be approached for the numerical integration results to be accepted. $\Delta\vartheta_1$ and $\Delta\vartheta_2$ represent the increment step of the summation variables $\vartheta_1$ and $\vartheta_2$, respectively.

To carry out the above integration we applied an extension of Simpson's rule to approximation of double integrals [9]. The results are shown in Fig. 2 as a function of the relative distance between coils $(\mathscr{D}/R)$.[1] Note that for negative values of $\mathscr{D}$, i.e., when the two loops do not overlap, the evaluated result consists of one value, since no infinites occur. The convergence threshold error was set to $10^{-5}$ and was satisfied for $\Delta\vartheta_1 = \Delta\vartheta_2 = u = 0.01$ rads. The exact value of $\mathscr{D}$ for which the magnetic flux reaches zero was found to be equal to $0.4783R$. This distance, which represents the solution of (6), is of great importance for the problem we are presenting here. It will be denoted as $D$ in what follows.

## IV. Experimental Results

We constructed two identical magnetic loops with radius $R = 10$ cm, using coaxial cable (RG 58) for electrostatic shielding. The coupling was measured by transmitting through one loop and receiving by the other. Both loops were terminated at 50 Ω, and a spectrum analyzer was used to measure the coupled energy. The whole assembly is shown in Fig. 3(a). The 0 dB level was set to the maximally induced energy.

The measurements confirmed the theoretical results presented in Section III for frequencies up to 10 MHz, as shown in Table I (column b). However, for frequencies higher than 10 MHz the distance of the minimum coupling $(\mathscr{D})$ augments. This is due to the effect of the capacitive coupling. This coupling, which is added to the mutual magnetic (inductive) coupling, depends on the geometry of the coils (common area) and the frequency. Its measured values are shown in Table I (column c). For frequencies higher than 50 MHz this coupling impedes the exact evaluation of a minimum coupling.

The effects of the capacitive coupling may be clarified if an equivalent circuit of the experimental setup is analyzed. A

[1] Refer to (5): assuming that $R = 1$ m and $I = 1$ A, then the values shown in Fig. 2 must be multiplied by $\mu/4\pi$ for the results to be in webers.

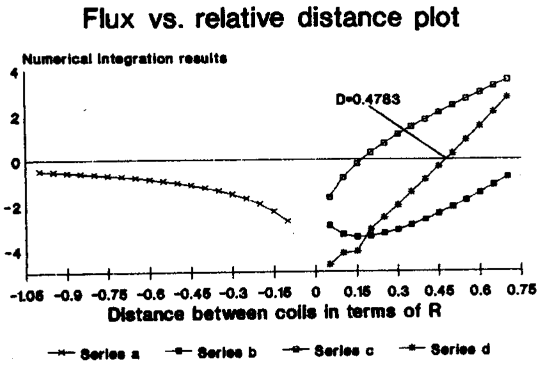


Fig. 2. The results of numerical integration for different values of $\mathcal{D}$. Series a represent $\Phi_2$ for negative values of $\mathcal{D}$. Series b represent the integral values of $0 \le \vartheta_2 \le x - u$ and series c the ones for $x + u \le \vartheta_2 \le \pi$—(7). Series d represent $\Phi_2$ for positive values of $\mathcal{D}$ ($d(\mathcal{D}) = b(\mathcal{D}) + c(\mathcal{D})$). Unit equivalences are explained in footnote 1.

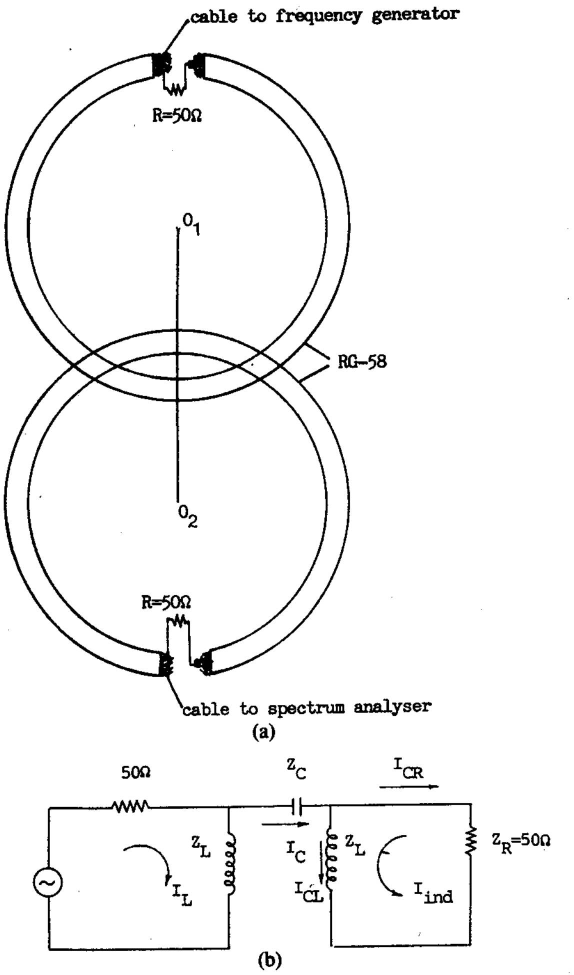


Fig. 3. The experimental setup. (a) Actual assembly. (b) Equivalent circuit. Notation: $Z_c$ represents the parasitic capacitive coupling. $Z_L$ denotes the inductive impedance of the coil and $Z_R$ the terminating resistance. $I_{ind}$ denotes the current due to inductive coupling, whereas $I_c$ denotes the current due to capacitive coupling. Voltage $V_{coup}$ is the one that is desired to be minimized.

first-order approximation of such a circuit is shown in Fig. 3(b). Further discussion on the effects and importance of the capacitive coupling is presented in section VI.

## V. The Problem of More Than Two Loops

Consider a number of $n$ loops with their centers lying on a straight line (Fig. 4). Each pair satisfies the condition for

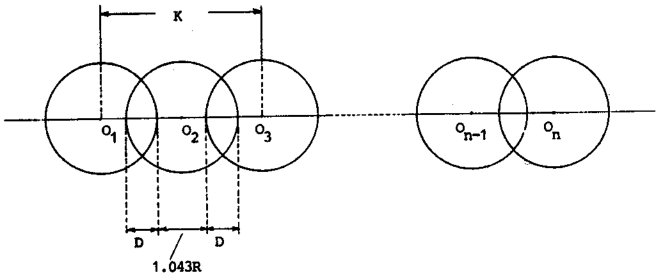


Fig. 4. A linear array antenna consisting of $n$ loops.

TABLE I
Experimental Results

| a | b | c | d | e |
|---|---|---|---|---|
| (MHz) | (cm) | (dB) | (dB) | (dB) |
| 5 | 4.8 | 0 | −38 | −22 |
| 10 | 4.8 | 0 | −38 | −22 |
| 15 | 4.9 | −70 | −38 | −22 |
| 20 | 5.1 | −70 | −36 | −22 |
| 25 | 5.3 | −68 | −35 | −22 |
| 30 | 5.5 | −58 | −34 | −21 |
| 35 | 6 | −46 | −33 | −20 |
| 40 | 6.9 | −34 | −32 | −17 |
| 45 | 7.6 | −30 | −27 | −16 |
| 50 | 8.1 | −28 | −32 | −18 |
| 55 | --- | −25 | −30 | −17 |

Column a presents the frequencies for which measurements were made. Column b contains the measured distance $D$, i.e., the distance where the coupling between the two loops was minimum. The measurement error is estimated to be ±0.5 cm. The theoretically predicted value of 4.783 cm (for radius of 10 cm) is satisfied for frequencies up to 10 MHz. Column c contains the residual coupling between the two loops, i.e., the coupling when the two loops are in orthogonal positions. Columns d and e contain the mutual coupling for the case $\mathcal{D} = R = 10$ cm and $\mathcal{D} = 0.15R = 1.5$ cm, respectively. In all cases the maximum coupling (which of course was achieved for $\mathcal{D} = -2R$, i.e., when the two loops were one over the other) has been set to 0 dB.

zero mutual magnetic coupling. However, there is an inevitable coupling between every two nonadjacent loops. The minimum distance between two such loops is $\mathcal{D} = 2(R - D) = 1.043R \simeq R$. This coupling has been measured experimentally and is shown in Table I (column d). The theoretically obtained value for the mutual flux is $-0.496$[1] (Fig. 2). Recall that in the numerical integration results scale, zero corresponds to zero coupling. Supposing that a coupling less than $10^{-2}$ (or $10^{-3}$ up to 40 MHz) is maintained we can use such a linear array antenna.

The addition of a new line as shown in Fig. 5(a) results in an inevitable magnetic coupling between loops diagonally adjacent. Two such loops have a distance $\mathcal{D} = 0.125R$, as can be verified by carrying out a few simple calculations. The measured coupling for such a distance is shown in Table I (column e). The theoretically obtained value for the mutual flux is $-2.003$[1] (Fig. 2).

The above-mentioned geometry corresponds to the "worst case" evaluation. The best geometrical position of the antenna elements for dense spatial coverage is shown in Fig. 5(b). In this case the minimum distance is again easily found to be $\mathcal{D} = 0.636R$. The theoretically obtained value for the mutual flux is $-0.822$[1] (Fig. 2). Such a geometry has been proposed for practical usage [10]. Whether or not it will be acceptable depends highly on the specifications of each application.

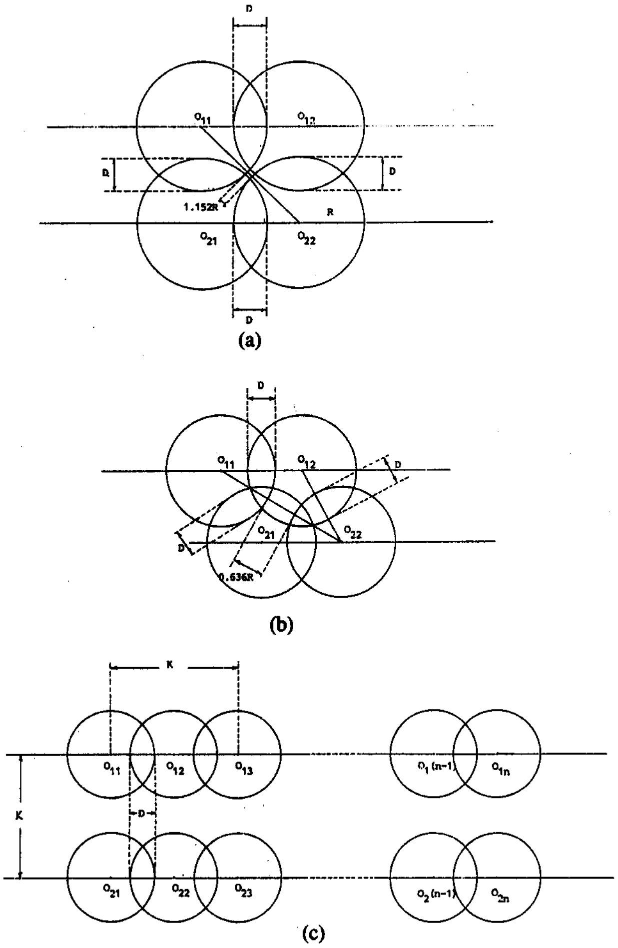


Fig. 5. Two-dimensional array antenna. (a) "Worst case" geometry. (b) Geometry that offers the most dense possible coverage. (c) Highest independence between two receiving points.

Alternatively, one may use successive linear array antennas at a distance satisfying the condition set for the single linear antenna earlier (Fig. 4). If this is the case, a low mutual coupling can be achieved. The array antenna is shown in Fig. 5(c). The distance condition results to $K = 2R + 2(R - D) \cong 3R$.

## VI. Discussion

We showed the conditions under which many antenna coils can be placed together in order to form an array antenna. We proved that two loops do not interact if a certain geometrical condition is satisfied.

Experimental results confirmed this condition, but showed that care must be taken for the minimization of capacitive coupling, if the antennas work at higher frequencies. Supposing that the length of the loops is small compared to the wavelength in use, appropriate shielding of the loop antennas can minimize the capacitive coupling to an acceptable level. Coaxial cables may be used for shielding purposes, as it was done in our experimental setup. However, use of solid transmission lines is expected to practically zero the capacitive coupling. Additionally, appropriate electrostatic shielding of the terminating resistor (Fig. 3a) should be employed.

Referring to Fig. 3(b), it is straightforward to show that the following equation holds

$$V_{coup} = Z_L(I_{CL} + I_{ind}) - Z_R I_{CR}$$

where $I_C = I_{CR} + I_{CL}$. Ideally, $V_{coup}$, i.e., the emf induced in the second loop, must equal zero. It depends highly on the particular case how it will be possible to fulfill this condition ($V_{coup} = 0$), since $V_{coup}$ is a function of $Z_L$, $Z_R$, and $I_C$. These variables have values that depend on the material and geometry of each coil and the relative positioning of them.

In conclusion, the capacitive coupling adds an "in-phase" component which results in a need for a geometrical displacement of the two coils to cancel it out by an equivalent "out of phase" induced current component. This explains the augmentation of the distance $\mathscr{D}$ according to the frequency. Therefore, the deviation from the theoretical noninteraction position gives a measure of the capacitive coupling of the loops.

The proposed two-dimensional array antenna (Fig. 5(b)) will not be appropriate in applications which require high independence between receiving points. On the other hand, the alternative geometry (Fig. 5(c)) can not be chosen in situations where dense spatial coverage is wanted. In most applications one of the two possibilities will offer an adequate solution.

## V. Conclusion

The geometrical arrangement of two circular loops for which zero mutual coupling is achieved has been established. Theoretical analysis and experimental measurements coincided quite satisfactorally. Thus, one can safely argue that a linear array as the one shown in Fig. 4 can be used in practice, provided the preceding discussion on the parasitic capacitive coupling will be taken into consideration.

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