Implementation of an extended fuzzy VIKOR method based on triangular and trapezoidal fuzzy linguistic variables and alternative deffuzification techniques

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Abstract. Many Multi-Criteria Decision Making (MCDM) problems contain information about the criteria and/or the alternatives that is either unquantifiable or incomplete. Fuzzy set theory has been successfully combined with MCDM methods to deal with imprecision. The fuzzy VIKOR method has been successfully applied in such problems. There are many extensions of this method; some of them utilize triangular fuzzy numbers, while others use trapezoidal fuzzy numbers. In addition, there are many defuzzification techniques available that are used in different variants. The use of each one of these techniques can have a substantial impact on the output of the fuzzy VIKOR method. Hence, we extend the fuzzy VIKOR method in order to allow the use of several defuzzification techniques. In addition, we allow the use of both triangular and trapezoidal fuzzy numbers. In this paper, we also present the implementation of a web-based decision support system that incorporates the fuzzy VIKOR method. Finally, an application of the fuzzy VIKOR method on a facility location problem is presented to highlight the key features of the implemented system.

Key words: multiple attribute decision making, VIKOR, fuzzy, decision support system, defuzzification

1 Introduction

Multi-Criteria Decision Making (MCDM) is a branch of operations research that can be applied for making complex decisions when many criteria are involved. MCDM methods are separated into Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM) [15] based on the determination of the alternatives. In MODM, alternatives are not predetermined but instead a set of objective functions is optimized subject to a set of constraints. In MADM, alternatives are predetermined and a limited number of alternatives is to be evaluated against a set of attributes. One of the most widely-used MADM methods is the VIKOR method [7]. The VIKOR method is considered to be effective in cases

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where the decision maker cannot be certain how to express his/her preferences coherently and consistently at the initial stages of the system design [8]. Yu [19] and Zeleny [23] provide the setting theory for compromise solutions. Opricovic and Tzeng [9] state that a compromise solution is a feasible solution, which is closest to the ideal, and compromise means an agreement established by mutual concessions. Hence, the compromise solution can serve as a basis for negotiations. The VIKOR method has been successfully applied on several application areas [18].

There are situations where decision makers have to deal with unquantifiable or incomplete information [22]. Fuzzy set theory can model imprecision in MCDM problems. Variants of the VIKOR method to determine a fuzzy compromise solution in MCDM problems with conflicting and noncommensurable criteria have been developed. In this paper, we utilize a fuzzy extension of the VIKOR method that is based on the methodology proposed by Sanayei et al. [13]; they use trapezoidal fuzzy numbers and in their paper, they focus on supplier selection, but the methodology can easily be applied in a broader scope as well. Variants of the fuzzy VIKOR method with triangular fuzzy numbers have been implemented in the past by Opricovic [10], Rostamzadeh et al. [12], Chen and Wang [2], and Wan et al. [17]. Variants using trapezoidal fuzzy numbers can be found in Shemshadi et al. [14], Ju and Wang [5], and Yucenur and Demirel [20].

In all variants of the fuzzy VIKOR method, a defuzzification technique is necessary to convert fuzzy numbers to crisp values. There are many defuzzification techniques proposed in the literature (for a literature review, see [16]). The use of each one of these techniques can have a substantial impact on the output of the fuzzy VIKOR method. Without trying to propose which method is best, we give the opportunity to decision makers to experiment with different methods and variations and decide which one fits their problem information. Hence, we extend the fuzzy VIKOR method proposed by Sanayei et al. [13] in order to allow the use of several defuzzification techniques. In addition, we allow the use of both triangular and trapezoidal fuzzy numbers. In this paper, we present the implementation of a web-based decision support system that incorporates the fuzzy VIKOR method. Decision makers can easily upload the input data and get illustrative results. Finally, an application of the fuzzy VIKOR method on a facility location problem is presented to highlight the key features of the implemented system.

The remainder of this paper is organized as follows. A background on fuzzy number theory is presented in Section 2. In addition, Section 2 details the defuzzification techniques that we incorporate in the fuzzy VIKOR method. Section 3 presents the fuzzy VIKOR method. In Section 4, the implemented decision support system is presented through a case study on a facility location problem. Finally, the conclusions of this paper are outlined in Section 5.

2 Background

A fuzzy set is a class with a continuum of membership grades [21]; thus, a fuzzy set A in a referential (universe of discourse) X is characterized by a membership function A, which associates with each element $x \in X$ a real number $A(x) \in [0,1]$, having the interpretation $A(x)$ as the membership grade of x in the fuzzy set A.

Let's consider now a fuzzy subset of the real line u: $R \rightarrow [0,1]$. u is a fuzzy number [1] [3], if it satisfies the following properties:

- u is normal, i.e., $\exists x_0 \in R$ with $u(x_0) = 1$.
- u is fuzzy convex, i.e., $u(tx+(1-t)y) \geq \min\{u(x), u(y)\}, \forall t \in [0,1], x, y \in R$.
- u is upper semi-continuous on R, i.e., $\forall \epsilon > 0, \exists \delta > 0$ such that $u(x) - u(x_0) < \epsilon, |x - x_0| < \delta$.
- u is compactly supported, i.e., $cl\{x \in R; u(x) > 0\}$ is compact, where $cl(A)$ denotes the closure of the set A.

The set of elements having the largest degree of membership in A is called the core of A:

$$\text{core}(A) = \{x | x \in X \text{ and } \neg(\exists y \in X)(A(y) > A(x))\} \tag{1}$$

The set of all elements that have a nonzero degree of membership in A is called the support of A:

$$\text{supp}(A) = \{x | x \in X \text{ and } A(x) > 0)\} \tag{2}$$

One of the most popular shapes of fuzzy numbers is the trapezoidal fuzzy number that can be defined as $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ with a membership function determined as follows (Figure 1 (b)):

$$\mu_A(x) = \begin{cases} 0, & x < \alpha_1 \\ \frac{x-\alpha_1}{\alpha_2-\alpha_1}, & \alpha_1 \leq x \leq \alpha_2 \\ 1, & \alpha_2 \leq x \leq \alpha_3 \\ \frac{\alpha_4-x}{\alpha_4}, & \alpha_3 \leq x \leq \alpha_4 \\ 0, & x > \alpha_4 \end{cases} \tag{3}$$

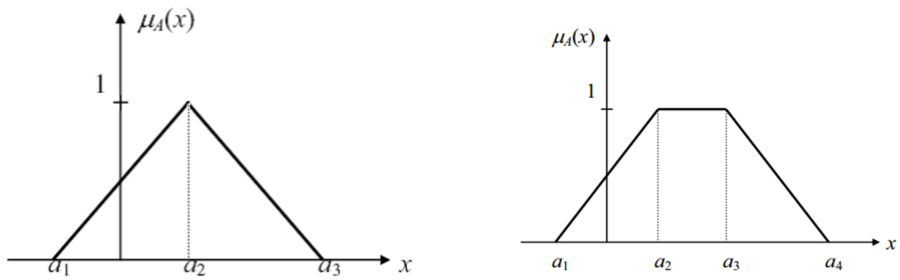
In the case where $\alpha_2 = \alpha_3$, the trapezoidal fuzzy number coincides with a triangular one (Figure 1 (a)).

Given a couple of positive trapezoidal fuzzy numbers $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $B = (b_1, b_2, b_3, b_4)$, the result of the addition and subtraction between trapezoidal fuzzy numbers is also a trapezoidal fuzzy number:

$$\begin{aligned} A(+)B &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4)(+)(b_1, b_2, b_3, b_4) \\ &= (\alpha_1 + b_1, \alpha_2 + b_2, \alpha_3 + b_3, \alpha_4 + b_4) \end{aligned} \tag{4}$$

and

$$\begin{aligned} A(-)B &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4)(-)(b_1, b_2, b_3, b_4) \\ &= (\alpha_1 - b_4, \alpha_2 - b_3, \alpha_3 - b_2, \alpha_4 - b_1) \end{aligned} \tag{5}$$



(a) Triangular fuzzy number (b) Trapezoidal fuzzy number

Fig. 1: Triangular (a) and trapezoidal (b) fuzzy numbers, adopted from Lee [6]

As for multiplication, division, and inverse, the result is not a trapezoidal fuzzy number.

A fuzzy vector is a vector that includes elements having a value between 0 and 1. Bearing this in mind, a fuzzy matrix is a gathering of such vectors. The operations on given fuzzy matrices $A = (\alpha_{ij})$ and $B = (b_{ij})$ are:

– sum

$$A + B = \max\left[\alpha_{ij}, b_{ij}\right] \tag{6}$$

– max product

$$A \cdot B = \max_k \left[\min\left(\alpha_{ik}, b_{kj}\right)\right] \tag{7}$$

– scalar product

$$\lambda A \tag{8}$$

where $0 \leq \lambda \leq 1$.

According to Zadeh [22], a linguistic variable is one whose values are words or sentences in a natural or artificial language. This kind of variables can well be represented by triangular or trapezoidal fuzzy numbers.

All defuzzification techniques can be formulated both in discrete and in continuous form. Without loss of generality and for simplicity, we will use the discrete formulation. The defuzzification techniques that are integrated into the proposed DSS are the following:

– First of maxima (FOM): FOM method selects the smallest element of the core of A as the defuzzification value:

$$FOM(A) = \min\left(\text{core}(A)\right) \tag{9}$$

- Last of maxima (LOM): LOM method selects the greatest element of the core of A as the defuzzification value:

$$LOM(A) = \max\left(\text{core}(A)\right) \tag{10}$$

- Middle of maxima (MOM): If the core of A contains an odd number of elements, then the middle element of the core is selected such that:

$$\left|\text{core}(A)_{<MOM(A)}\right| = \left|\text{core}(A)_{>MOM(A)}\right| \tag{11}$$

  If the core of A contains an even number of elements, then we can select an element as the deffuzification value such that:

$$\left|\text{core}(A)_{<MOM(A)}\right| = \left|\text{core}(A)_{>MOM(A)}\right| \pm 1 \tag{12}$$

- Center of gravity (COG): COG method calculates the center of gravity of the area under the membership function:

$$\frac{\sum_{x_{min}}^{x_{max}} x\mu_A(x)}{\sum_{x_{min}}^{x_{max}} \mu_A(x)} \tag{13}$$

- Mean of maxima (MeOM): MeOM method is a variant of COG method. It computes the mean of all the elements of the core of A:

$$MeOM(A) = \frac{\sum_{x\in\text{core}(A)} x}{|\text{core}(A)|} \tag{14}$$

- Basic defuzzification distributions (BADD): BADD method [4] is an extension of the COG method. The defuzzification value is computed as follows:

$$BADD(A) = \frac{\sum_{x_{min}}^{x_{max}} x\mu_A^{\gamma}(x)}{\sum_{x_{min}}^{x_{max}} \mu_A^{\gamma}(x)} \tag{15}$$

  where γ is a free parameter in $[0, \infty)$. The parameter γ is used to adjust the method to the following special cases:

$$\begin{cases} BADD(A) = MeOS(A), & \text{if } \gamma = 0 \\ BADD(A) = COG(A), & \text{if } \gamma = 1 \\ BADD(A) = MeOM(A), & \text{if } \gamma \to \infty \end{cases} \tag{16}$$

  where $MeOS(A)$ is the mean of support of the core A.

3 The fuzzy VIKOR method

In this paper, we use a fuzzy extension of the VIKOR method that is based on the methodology proposed by Sanayei et al. [13]. This method uses trapezoidal fuzzy numbers and deffuzzifies the fuzzy decision matrix and fuzzy weight of each criterion into crisp values using the COG defuzzification technique (Equation (13)). However, we extend this method by allowing both triangular and trapezoidal fuzzy numbers. In addition, decision makers can also use any of the defuzzification techniques presented in the previous section.

Let us assume that an MADM problem has m alternatives, $A_1, A_2, \cdots, A_m$, and n decision criteria, $C_1, C_2, \cdots, C_n$. Each alternative is evaluated with respect to the n criteria. All the alternative evaluations form a decision matrix $X = (x_{ij})_{m \times n}$. Let $W = (w_1, w_2, \cdots, w_n)$ be the vector of the criteria weights, where $\sum_{j=1}^{n} w_j = 1$.

For simplicity, the steps of the method are presented using trapezoidal fuzzy numbers. The steps of the procedure are:

Step 1. Identify the objectives of the decision making process and define the problem scope

The decision goals and the scope of the problem are defined. Then, the objectives of the decision making process are identified.

Step 2. Arrange the decision making group and define and describe a finite set of relevant attributes

We form a group of decision makers to identify the criteria and their evaluation scales.

Step 3. Identify the appropriate linguistic variables

Choose the appropriate linguistic variables for the importance weights of the criteria and the linguistic ratings for the alternatives with respect to the criteria.

Step 4. Pull the decision maker opinions to get the aggregated fuzzy weight of criteria and aggregated fuzzy rating of alternatives and construct a fuzzy decision matrix

Let the fuzzy rating and importance weight of the kth decision maker be $\widetilde{x}_{ijk} = (\widetilde{x}_{ijk1}, \widetilde{x}_{ijk2}, \widetilde{x}_{ijk3}, \widetilde{x}_{ijk4})$ and $\widetilde{w}_{ijk} = (\widetilde{w}_{ijk1}, \widetilde{w}_{ijk2}, \widetilde{w}_{ijk3}, \widetilde{w}_{ijk4})$, respectively, where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. Hence, the aggregated fuzzy ratings $(\widetilde{x}_{ij})$ of alternatives with respect to each criterion can be calculated as:

$$\widetilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}) \tag{17}$$

where:

$$\begin{aligned} x_{ij1} &= \min_k \{x_{ijk1}\}, x_{ij2} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk2}, \\ x_{ij3} &= \frac{1}{K}\sum_{k=1}^{K} x_{ijk3}, x_{ij4} = \max_k \{x_{ijk4}\} \end{aligned} \tag{18}$$

The aggregated fuzzy weights $(\widetilde{w}_j)$ of each criterion can be calculated as:

$$\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4}) \tag{19}$$

where:

$$w_{j1} = \min_k \{w_{jk1}\}, w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2},$$
$$w_{j3} = \frac{1}{K}\sum_{k=1}^{K} w_{jk3}, w_{ij4} = \max_k \{w_{jk4}\} \tag{20}$$

The problem can be concisely expressed in matrix format as follows:

$$\widetilde{D} = \begin{bmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn} \end{bmatrix} \tag{21}$$

and the vector of the criteria weights as:

$$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \cdots, \widetilde{w}_n] \tag{22}$$

where $\widetilde{x}_{ij}$ and $\widetilde{w}_j, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, are linguistic variables according to Step 3. They can be approximated by the trapezoidal fuzzy numbers $\widetilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4})$ and $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$.

Step 5. Defuzzify the fuzzy decision matrix and fuzzy weight of each criterion into crisp values

Deffuzzify the fuzzy decision matrix and fuzzy weight of each criterion into crisp values using any of the defuzzification techniques presented in the previous section.

Step 6. Determine the best and the worst values of all criteria functions

Determine the best f_j^* and the worst f_j^- values of all criteria functions:

$$f_j^* = \max_i f_{ij}, f_j^- = \min_i f_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{23}$$

if the jth function is to be maximized (benefit) and:

$$f_j^* = \min_i f_{ij}, f_j^- = \max_i f_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{24}$$

if the jth function is to be minimized (cost).

Step 7. Compute the values S_i and R_i

Compute the values S_i and R_i using the relations:

$$S_i = \sum_{j=1}^{n} w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right), i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{25}$$

$$R_i = \max_j \left[w_j \left(f_j^* - f_{ij}\right) / \left(f_j^* - f_j^-\right)\right], i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{26}$$

Step 8. Compute the values Q_i

Compute the values Q_i using the relation:

$$Q_i = v\left(S_i - S^*\right)/\left(S^- - S^*\right) + (1-v)\left(R_i - R^*\right)/\left(R^- - R^*\right), \quad i = 1, 2, \cdots, m \tag{27}$$

where $S^* = \min_i S_i$; $S^- = \max_i S_i$; $R^* = \min_i R_i$; $R^- = \max_i R_i$; and v is introduced as a weight for the strategy of the "maximum group utility", whereas $1 - v$ is the weight of the individual regret.

Step 9. Rank the alternatives

Rank the alternatives, sorting by the values S, R, and Q in ascending order. The results are three ranking lists.

Step 10. Propose a compromise solution

Propose as a compromise solution the alternative $[A^{(1)}]$, which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

– C1 - Acceptable advantage

$$Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geq DQ \tag{28}$$

where $A^{(2)}$ is the second ranked alternative by the measure Q and $DQ = 1/(m-1)$.

– C2 - Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by S and/or R. This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility ($v > 0.5$), or "by consensus" ($v \approx 0.5$), or "with veto" ($v < 0.5$). If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:
  – Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
  – Alternatives $A^{(1)}, A^{(2)}, \cdots, A^{(l)}$ if the condition C1 is not satisfied; $A^{(l)}$ is determined by the relation $Q(A^{(l)}) - Q(A^{(1)}) < DQ$ for maximum l (the positions of these alternatives are "in closeness").

4 Presentation of the decision support system on a facility location problem

The web-based decision support system has been implemented using PHP, MySQL, Ajax, and jQuery. In [11], we presented a DSS that included TOPSIS and VIKOR in a nonfuzzy environment. We extend the DSS to allow decision makers solve group decision-making MADM problems in a fuzzy environment. Next, we present the steps that the decision maker should perform in order to solve a group decision-making MADM problem. More specifically, we consider the facility location (or location - allocation problem). The facility location problem is a well known and extensively studied problem in the operational research discipline. In this case study, a firm is trying to identify the best site out of ten possible choices in order to locate a production facility, taking in the same time into account four criteria: (i) the investment costs, (ii) the employment needs, (iii) the social impact, and (iv) the environmental impact. The first two criteria need to be minimized, while the latter two need to be maximized. The importance weights of the qualitative criteria and the ratings are considered as linguistic variables expressed in positive trapezoidal fuzzy numbers, as shown in Table 1.

Table 1: Linguistic variables for the criteria

Linguistic variables for the importance weight of each criterion		Linguistic variables for the ratings	
Very low (VL)	(0, 0, 0.1, 0.2)	Very poor (VP)	(0.0, 0.0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.2, 0.3)	Poor (P)	(0.1, 0.2, 0.2, 0.3)
Medium low (ML)	(0.2, 0.3, 0.4, 0.5)	Medium poor (MP)	(0.2, 0.3, 0.4, 0.5)
Medium (M)	(0.4, 0.5, 0.5, 0.6)	Fair (F)	(0.4, 0.5, 0.5, 0.6)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)	Medium good (MG)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.8, 0.9)	Good (G)	(0.7, 0.8, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1.0, 1.0)	Very good (VG)	(0.8, 0.9, 1.0, 1.0)

We assume that we have already formed a group of decision makers and one of them acts as the leader of the group. Initially, the leader creates a new MADM problem in the DSS. He/she should enter the following information:

- the name and type (benefit or cost) of each criterion and the name of each alternative (Figure 2).
- the type of fuzzy numbers (triangular or trapezoidal), the deffuzification technique, and the value of the maximum group utility strategy (v). In addition, the decision maker provides the linguistic variables for the criteria and the alternatives (Figure 3). The center of gravity defuzzification technique along with trapezoidal fuzzy numbers is used in this case study.

Criteria and alternatives are considered to be evaluated by decision makers that are experts on the field. The evaluations of four decision makers are in Tables 2 and 3. Each decision maker evaluates the criteria and alternatives using a linguistic variable (Figure 4).

Table 2: The importance weight of the criteria for each decision maker

	D_1	D_2	D_3	D_4
Investment costs	H	VH	VH	H
Employment needs	M	H	VH	H
Social impact	M	MH	ML	MH
Environmental impact	H	VH	MH	VH

When all decision makers have entered their evaluations, the leader can see the results of the fuzzy VIKOR method. The results are graphically and numerically displayed (Figure 5). The DSS can also output a thorough report in a pdf

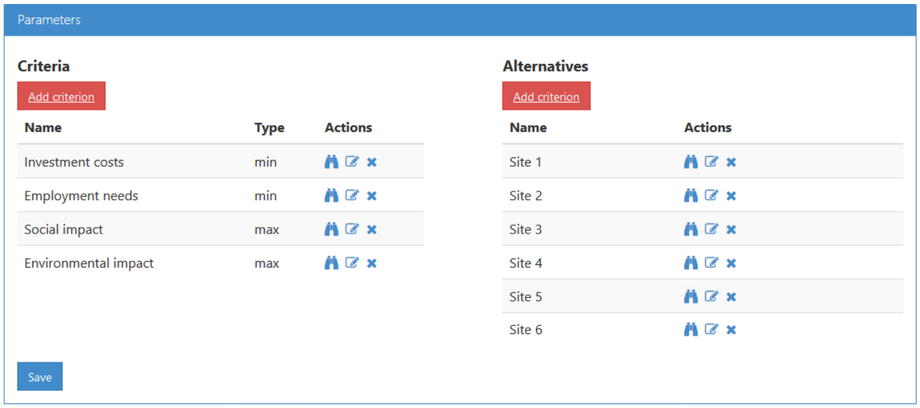


Fig. 2: Defining criteria and alternatives

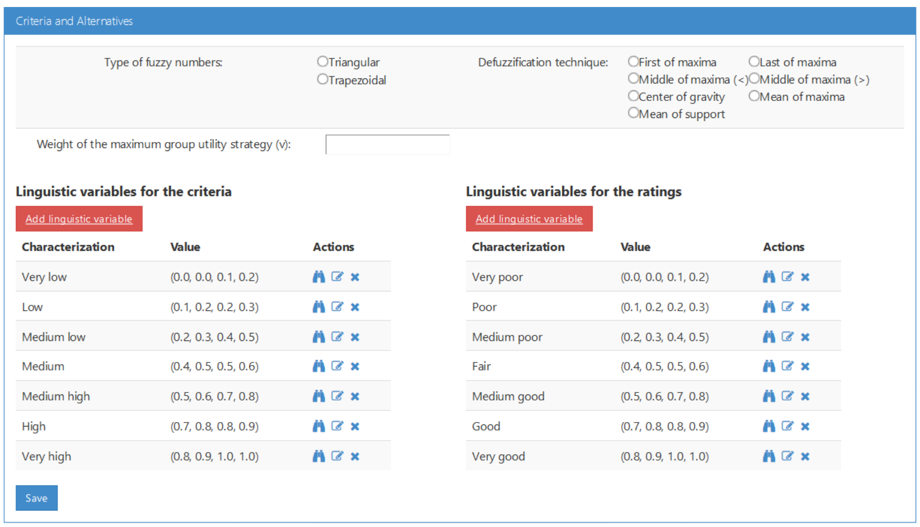


Fig. 3: Defining linguistic variables for criteria and alternatives and algorithmic parameters

file containing the results of the fuzzy VIKOR method. The result is a compromise solution (if the acceptable advantage condition (C1) and the acceptable stability condition (C2) are met) or a set of compromise solutions.

The final results are shown in Figure 5. The ranking according to the measure S is the following (first is the most preferred site):

$$8 - 7 - 3 - 5 - 2 - 6 - 1 - 10 - 9 - 4$$

Table 3: The ratings of the ten sites by the four decision makers for the four criteria

Criteria	Candidate sites	Decision makers				Criteria	Candidate sites	Decision makers			
		D_1	D_2	D_3	D_4			D_1	D_2	D_3	D_4
Investment costs	Site 1	VG	G	MG	MG	Employment needs	Site 1	F	MG	MG	G
	Site 2	MP	F	F	P		Site 2	F	VG	G	G
	Site 3	MG	MP	F	F		Site 3	MG	MG	VG	G
	Site 4	MG	VG	VG	MG		Site 4	G	G	VG	G
	Site 5	VP	P	G	P		Site 5	P	VP	MP	MP
	Site 6	F	G	G	G		Site 6	F	MP	MG	MG
	Site 7	P	P	G	P		Site 7	VP	P	VP	MP
	Site 8	F	F	F	F		Site 8	VG	G	F	G
	Site 9	VG	VG	MG	MG		Site 9	P	F	VP	VP
	Site 10	VG	VG	VG	VG		Site 10	VG	G	VG	G
Social impact	Site 1	P	P	MP	MP	Environmental impact	Site 1	G	VG	G	G
	Site 2	MG	VG	G	VG		Site 2	MG	F	MP	F
	Site 3	MP	F	F	F		Site 3	MP	P	P	P
	Site 4	MG	VG	G	VG		Site 4	VP	F	P	F
	Site 5	G	G	VG	G		Site 5	G	MG	MG	MG
	Site 6	VG	MG	F	F		Site 6	P	MP	F	F
	Site 7	G	VG	VG	VG		Site 7	VG	MG	MG	G
	Site 8	MG	MG	G	G		Site 8	F	MP	F	G
	Site 9	VG	G	VG	VG		Site 9	G	G	VG	VG
	Site 10	VP	F	F	VP		Site 10	G	G	VG	VG

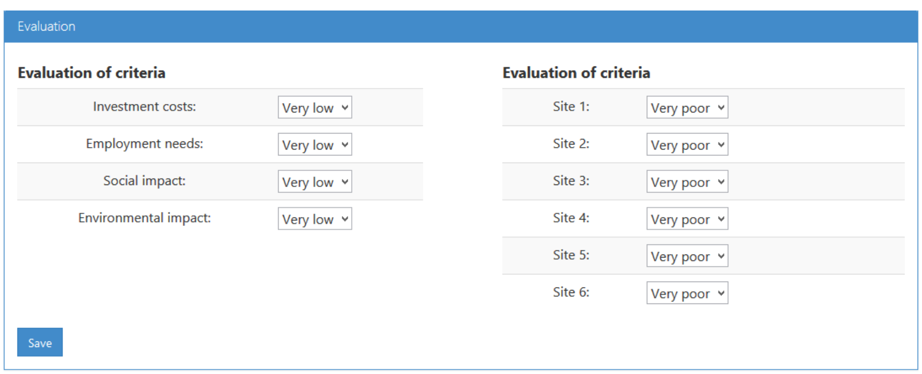


Fig. 4: Evaluating criteria and alternatives

Similarly, the ranking according to the measure R is the following:

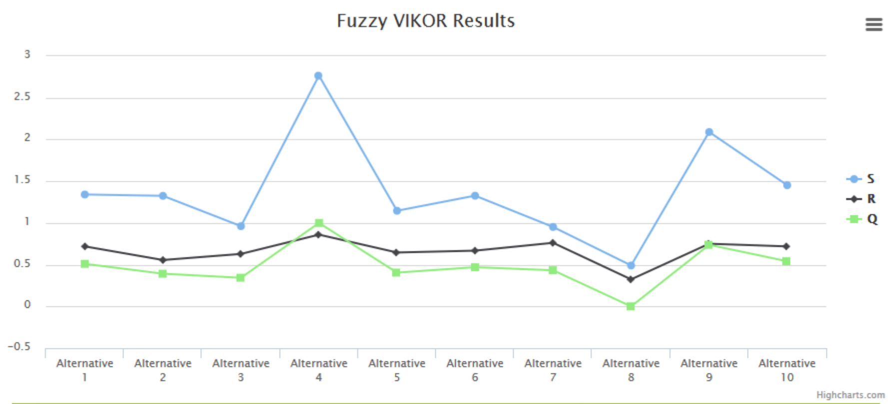
$$8 - 2 - 3 - 5 - 6 - 1 - 10 - 9 - 7 - 4$$

Finally, the ranking according to the measure Q is the following:

$$8 - 3 - 2 - 5 - 7 - 6 - 1 - 10 - 9 - 4$$

The best ranked alternative is Site 8 since it satisfies the conditions C1 and C2:

- C1: $Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geq DQ \Rightarrow 0.343 - 0 \geq 0.11$ that holds true.
- Site 8 is best ranked by measures S, R, and Q.



	Alternatives										Ranking
	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	
S	1.341	1.325	0.962	2.765	1.146	1.327	0.952	0.491	2.090	1.453	A8, A7, A3, A5, A2, A6, A1, A10, A9, A4
R	0.719	0.556	0.629	0.860	0.646	0.668	0.762	0.324	0.751	0.719	A8, A2, A3, A5, A6, A1, A10, A9, A7, A4
Q	0.510	0.392	0.343	1.000	0.405	0.470	0.433	0.000	0.738	0.541	A8, A3, A2, A5, A7, A6, A1, A10, A9, A4

Compromise solution: Alternative 8

Fig. 5: Final results for the facility location problem

5 Conclusions

A common problem in many MCDM problems is the existence of unquantifiable or incomplete information about the criteria and/or the alternatives. One way to deal with imprecision is the utilization of fuzzy set theory in the traditional MCDM methods. The fuzzy VIKOR method is gaining popularity in such problems. However, there are many extensions of this method. Some extensions utilize triangular fuzzy numbers, while others use trapezoidal fuzzy numbers. In addition, there are many defuzzification techniques that are used in different variants. In all variants of the fuzzy VIKOR method, a defuzzification technique is necessary to convert fuzzy numbers to crisp values. The use of a defuzzification technique in the fuzzy VIKOR method can have a substantial impact on its output. Hence, it is critical to allow decision makers experiment with different defuzzification techniques.

Without trying to propose which method is best, we give the opportunity to decision makers to experiment with different methods and variations and decide which one fits their problem information. Hence, we extended the fuzzy VIKOR method proposed by Sanayei et al. [13] in order to allow the use of several defuzzification techniques. In addition, we allow the use of both triangular and trapezoidal fuzzy numbers. We also presented the implementation of a web-based decision support system that incorporates the fuzzy VIKOR method. Decision makers can easily upload the input data and get thorough illustrative results. Finally, an application of the fuzzy VIKOR method on a facility location problem was presented to highlight the key features of the implemented system.

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