A decision support system for multiple criteria alternative ranking using TOPSIS and VIKOR in fuzzy and nonfuzzy environments

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Abstract

TOPSIS and VIKOR are two well-known and widely-used multiple attribute decision making methods. Many extensions of these methods have been proposed that either use different techniques to rank alternatives or utilize fuzzy logic to handle alternatives and criteria that are unquantifiable and/or incomplete. In this paper, we present the implementation of a web-based decision support system that incorporates TOPSIS and VIKOR to solve multicriteria decision making problems either in a nonfuzzy or in a fuzzy environment. The aim of this paper is to present a tool that will be used by decision makers to compare various alternative solutions and understand how robust a decision will be. The proposed system can be used both in single and in group decision making problems. In addition, we review several variations for each step of these methods and implement different techniques when applicable. Hence, decision makers can experiment with different techniques in each method. We implement ten normalization techniques, three methods for the calculation of the ideal and anti-ideal solutions, fifteen distance metrics, five fuzzy distance metrics, and eight defuzzification techniques. An illustrative example is presented to highlight the key features of the implemented system and the different scenarios that

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can be built using the proposed DSS.

Keywords: Multiple attribute decision making, Fuzzy aggregation operators, Defuzzification, Fuzzy distance metrics, Group decision making

1. Introduction

Multi-Criteria Decision Making (MCDM) is a well-known field of operations research and the most well-known branch of decision making. It is a branch of a general class of operation research models that can be applied for complex decisions when a lot of criteria are involved. MCDM methods are divided into Multi-Objective Decision Making (MODM) and Multi-Attribute Decision Making (MADM) [80]. The major difference of these families of methods is based on the determination of the alternatives. In MODM, the alternatives are not predetermined but instead a set of objective functions is optimized subject to a set of constraints. In MADM, the alternatives are predetermined and a limited number of alternatives is to be evaluated against a set of attributes. Well-known MODM methods include bounded objective function formulation, genetic algorithms, global criterion formulation, and goal programming, while well-known MADM methods include AHP, ELECTRE, PROMETHEE, TOPSIS, and VIKOR.

TOPSIS and VIKOR are well-known and pretty straightforward MADM methods. TOPSIS is is based on finding an ideal and an anti-ideal solution and comparing the distance of each one of the alternatives to those. On the other hand, VIKOR has been developed to provide compromise solutions to discrete multiple criteria problems that include non-commensurable and conflicting criteria. Both methods are based on an aggregating function representing the closeness to the ideal. TOPSIS and VIKOR have attracted much attention among researchers and many variants and extensions of these methods have been proposed. Regarding different methods in the various steps of TOPSIS, researchers focused on the normalization procedure [8, 41, 54, 81, 93], the determination of the ideal and the anti-ideal solution [20, 30], and the distance metric used to calculate the distance of each alternative from the ideal and anti-ideal solutions [12, 56, 64]. On the contrary, the methods used in the various steps of VIKOR have not been modified/extended. However, Opricovic & Tzeng [60, 61] extended at a later stage VIKOR with four new steps which provided a stability analysis to determine the weight stability intervals and included a trade-off analysis.

There are situations where decision makers have to deal with unquantifiable or incomplete information [89]. In real-world problems, available data can very often be incomplete, non-obtainable or imprecise, and as such not deterministic. Therefore, data can well be fuzzy, a situation where linguistic variables can depict the decision makers preferences in a more appropriate way. Then, the assessment of alternatives for each criterion and the weights associated to the criteria are suitable for the use of linguistic variables rather than numerical ones. The concept of linguistic variables is therefore very useful when the analyst needs to describe complex situations or sometimes not well defined by conventional quantitative expressions. As an example, the notion of criteria weight if represented by such a linguistic variable can have the values of very low, low, medium, high, very high, or something similar. It is also easier for the decision maker to deal with linguistic variables instead of their deterministic counterparts when trying to reach for a decision.

Fuzzy set theory can model imprecision in MADM problems. The diffusion of fuzzy set theory into MADM methods has created a new decision theory paradigm, known as fuzzy MADM, that is widely-used in decision making problems. TOPSIS and VIKOR were further extended to handle fuzzy numbers involving the opinions of a number of independent experts. Regarding different methods in the various steps of fuzzy TOPSIS, there are many fuzzy TOPSIS extensions focusing on the distance measurement [9, 14, 27, 50], the calculation of the ideal and anti-ideal points [14, 50], and the use of different type of fuzzy numbers, like triangular [14, 23, 83] and trapezoidal [46, 50]. On the other hand, researchers modified fuzzy VIKOR by utilizing different defuzzification techniques [2, 57, 76] and the use of different type of fuzzy numbers, like triangular [57, 73, 76] and trapezoidal [36, 75, 88].

The selection of the most suitable MADM method for a specific problem is a difficult task. There are many factors that should be considered before selecting an MADM method or a combination of MADM methods. Guitouni & Martel [31] proposed a conceptual framework for articulating tentative guidelines to choose an appropriate MADM method. Roy & Slowiński [74] presented a general framework to guide decision makers in choosing the right method for a specific problem. Kurka & Blackwood [38] provided a methodology to systematically select an MADM method in the energy and renewable energy sectors. Zanakis et al. [90] compared the performance of eight MADM methods, namely ELECTRE, MEW, SAW, TOPSIS, and four versions of AHP. They found out that the final rankings of the alternatives vary across methods, especially in problems with many alternatives. Opricovic & Tzeng [59] presented a comparative analysis of TOPSIS and VIKOR in order to show their similarities and differences. The analysis revealed that TOPSIS and VIKOR use different normalization techniques and that they introduce different aggregating functions for ranking. Opricovic & Tzeng [60] compared the extended VIKOR method with ELECTRE II, PROMETHEE, and TOPSIS. Ranking results were similar for ELECTRE II, PROMETHEE, and VIKOR. Chu et al. [21] presented a comparison of SAW, TOPSIS, and VIKOR. They found out that TOPSIS and SAW had identical rankings, while VIKOR produced different rankings. They concluded that both TOPSIS and VIKOR are suitable for assessing similar problems and provide results close to reality. Ertuğrul & Karakaşoğlu [28] compared fuzzy AHP and fuzzy TOPSIS. They applied these methodologies to facility location selection problem finding the same ranking with both methods. Hajkowicz & Higgins [32] compared the weighted summation, range of value, PROMETHEE II, Evamix, and compromise programming methodologies. They concluded that different multicriteria methods were in strong agreement with high correlations amongst rankings. Ozcan et al. [62] compared the AHP, TOPSIS, ELECTRE, and Grey Theory methods and detailed the advantages and disadvantages of them. They found out that TOPSIS and ELECTRE produce similar results.

A common problem is that different MADM methods result to different ranking results. Hence, many researchers apply different MADM methods and compare the corresponding rankings. Even using different techniques in a step of a single MADM method may result in different results. The aim of this paper is threefold. Firstly, we review different variants of the TOPSIS and VIKOR methods focusing on variants used both in a nonfuzzy and in a fuzzy environment. Several techniques for each of these steps are reviewed. More specifically, we review ten normalization techniques, three methods for the calculation of the ideal and anti-ideal solutions, fifteen distance metrics, five fuzzy distance metrics, and eight defuzzification techniques. This is the first time that most of these methods are incorporated into TOPSIS and VIKOR. Secondly, even though there are many recent papers (e.g., [3, 39, 55, 71, 86]) that compare TOPSIS and VIKOR methods in various applications, most of these techniques have not been utilized in MADM techniques yet. Hence, we present various normalization techniques, distance metrics, and defuzzification techniques that can be used in TOPSIS, VIKOR, and their fuzzy variants. Finally, we implement a web-based decision support system that incorporates all these MADM methods and variants. To the best of our knowledge, this is the first DSS that provides decision makers with so many different extensions of TOPSIS, VIKOR, and their fuzzy extensions. The decision makers can apply different techniques in various steps of the TOPSIS and VIKOR methods and compare the results. In addition, they can use the proposed DSS in a group decision making environment and find optimal solutions without directly interacting with each other. The overall goal of this paper is to present all alternative methods that can be utilized in TOPSIS, VIKOR, and their fuzzy extensions. Selecting a suitable MADM method for a specific case study is challenging [31, 38, 74]. If we also consider the alternative methods that can be used in MADM methods, then the complexity of this task increases significantly. As it has been mentioned in previous works (e.g., [31]), it is very important to use a tool to choose an appropriate MADM method. Hence, we aim to present such a tool that will be used by decision makers to compare various alternative solutions and find out the most appropriate one for their specific case study.

In this paper, we present the implementation of a web-based decision support system that incorporates TOPSIS and VIKOR and allows decision makers to compare the results obtained from both methods. TOPSIS and VIKOR have been included in the DSS because they share the same theoretical background and the same input matrix; as such the results can be considered comparable. This paper is an extension of our previous works [63, 70], where we presented a decision support system with a limited number of different methods and techniques included. In the proposed DSS, decision makers can easily upload the input data and get thorough illustrative results. Different techniques are available for each step of these methods and decision makers can select them to obtain rankings according to a case's needs. The proposed paper is a significantly extended work. First of all, we have extended our previous works to allow multiple decision makers solve multicriteria group decision making problems. Secondly, decision makers can experiment with different techniques in each method and build several scenarios. We implement ten normalization techniques, three methods for the calculation of the ideal and anti-ideal solutions, fifteen distance metrics, five fuzzy distance metrics, and eight defuzzification techniques. The aim of this paper is not to compare analytically all different methods that can be utilized in each step of TOPSIS, VIKOR, and their fuzzy extensions. We do not intend to make suggestions about which method is better. Our aim is to provide decision makers with a robust tool to enable them to structure the problem according to their exact requirements. They can build the desired multiple criteria model and explore the various possibilities that arise from the use of different methods in each step of the TOPSIS and VIKOR methodologies. At a second stage, they can compare graphically the associated solutions, produce scenarios and maybe revise the original model in order to accommodate any needed changes. In this way, they can fine-tune the model and find more appropriate solutions. Additionally, as a byproduct, the DSS can well be used as a teaching tool. If the results obtained by using different methods are similar, this fact may be considered as a good indication that the proposed solution is optimal. In the opposite case, additional analysis of the criteria and their ranking is advised [27].

The remainder of this paper is organized as follows. A background on fuzzy number theory is presented in Section 2. We review TOPSIS, VIKOR, fuzzy

TOPSIS, and fuzzy VIKOR in Sections 3 and 4. For each method, we present different techniques that can be utilized in each step of these methods. We also present a literature review on comparative studies of such methods. In Section 5, the implemented decision support system is presented. Section 6 presents an illustrative example to highlight the key features of the implemented system and the different scenarios that can be built using the proposed DSS. Finally, the conclusions of this paper are outlined in Section 7.

2. Background

Let us assume that an MADM problem has m alternatives, $A_1, A_2, \cdots, A_m$, and n decision criteria, $C_1, C_2, \cdots, C_n$. Each alternative is evaluated with respect to the n criteria. All the alternatives evaluations form a decision matrix $X = (x_{ij})_{m \times n}$. Let $W = (w_1, w_2, \cdots, w_n)$ be the vector of the criteria weights, where $\sum_{j=1}^{n} w_j = 1$.

The goal of an MADM method is to rank the alternatives and find the best solution. Initially, the decision maker defines the criteria and the alternatives. Then, he/she sets values for the criteria weights and evaluates the alternatives with respect to the n criteria. The input of an MADM method is the decision matrix X and the weight vector w, while the output is a ranking of the alternatives.

However, there are situations where decision makers have to deal with unquantifiable or incomplete information [89]. Fuzzy set theory can model imprecision in MADM problems. Hence, variants of the "traditional" MADM methods have been developed to cope with unquantifiable or incomplete information. In these methods, a group of decision makers evaluate the criteria weights and the decision matrix using fuzzy numbers, especially triangular and trapezoidal fuzzy numbers. In the rest of this section, we introduce the notation that will be used to describe the fuzzy TOPSIS and VIKOR methods.

A fuzzy set is a class with a continuum of membership grades [89]; thus, a fuzzy set A in a referential (universe of discourse) X is characterized by a membership function A, which associates with each element $x \in X$ a real number $A(x) \in [0,1]$, having the interpretation $A(x)$ as the membership grade of x in the fuzzy set A.

Let's consider now a fuzzy subset of the real line u: $R \to [0,1]$. u is a fuzzy number [4, 25], if it satisfies the following properties:

- u is normal, i.e., $\exists x_0 \in R$ with $u(x_0) = 1$.
- u is fuzzy convex, i.e., $u(tx+(1-t)y) \geq \min\{u(x), u(y)\}, \forall t \in [0,1], x, y \in R$.
- u is upper semi-continuous on R, i.e., $\forall \epsilon > 0, \exists \delta > 0$ such that $u(x) - u(x_0) < \epsilon, |x - x_0| < \delta$.
- u is compactly supported, i.e., $cl\{x \in R; u(x) > 0\}$ is compact, where $cl(A)$ denotes the closure of the set A.

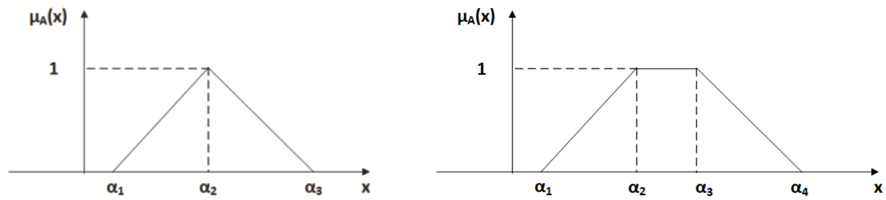
One of the most popular shapes of fuzzy numbers is the trapezoidal fuzzy number that can be defined as $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ with a membership function determined as follows (Figure 1 (b)):

$$\mu_A(x) = \begin{cases} 0, & x < \alpha_1 \\ \frac{x-\alpha_1}{\alpha_2-\alpha_1}, & \alpha_1 \leq x \leq \alpha_2 \\ 1, & \alpha_2 \leq x \leq \alpha_3 \\ \frac{\alpha_4-x}{\alpha_4}, & \alpha_3 \leq x \leq \alpha_4 \\ 0, & x > \alpha_4 \end{cases} \tag{1}$$

In the case where $\alpha_2 = \alpha_3$, the trapezoidal fuzzy number coincides with a triangular one (Figure 1 (a)).

Given a couple of positive trapezoidal fuzzy numbers $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and $B = (b_1, b_2, b_3, b_4)$, the result of the addition and subtraction between trapezoidal fuzzy numbers is also a trapezoidal fuzzy number:

$$\begin{aligned} A(+)B &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4)(+)(b_1, b_2, b_3, b_4) \\ &= (\alpha_1 + b_1, \alpha_2 + b_2, \alpha_3 + b_3, \alpha_4 + b_4) \end{aligned} \tag{2}$$



(a) Triangular fuzzy number (b) Trapezoidal fuzzy number

Figure 1: Triangular (a) and trapezoidal (b) fuzzy numbers

and

$$\begin{aligned} A(-)B &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4)(-)(b_1, b_2, b_3, b_4) \\ &= (\alpha_1 - b_4, \alpha_2 - b_3, \alpha_3 - b_2, \alpha_4 - b_1) \end{aligned} \tag{3}$$

A fuzzy vector is a certain vector that includes an element and has a value between 0 and 1. Bearing this in mind, a fuzzy matrix is a gathering of such vectors. The operations on given fuzzy matrices $A = (\alpha_{ij})$ and $B = (b_{ij})$ are:

- maximum

$$A + B = \max\left[\alpha_{ij}, b_{ij}\right] \tag{4}$$

- max-min composition

$$A \cdot B = \max_k\left[\min\left(\alpha_{ik}, b_{kj}\right)\right] \tag{5}$$

- scalar product

$$\lambda A = \lambda \times \left[\alpha_{ij}\right] \tag{6}$$

where $0 \leq \lambda \leq 1$.

According to Zadeh [89], a linguistic variable is one whose values are words or sentences in a natural or artificial language; among others, he provides an example in the form of the linguistic variable 'Age' that can take the values young, not young, very young, quite young, old, not very old and not very young, etc., rather than 20, 21, 22, 23, $\cdots$. Linguistic variables well be replaced by triangular or trapezoidal fuzzy numbers. Lee [44] further denotes that a linguistic variable can be defined by the quintuple

$$\text{Linguistic variable} = (x, T(x), U, G, M) \tag{7}$$

where:

- x: name of variable.
- $T(x)$: set of linguistic terms that can be a value of the variable.
- U: set of universe of discourse, which defines the characteristics of the variable.
- G: syntactic grammar that produces terms in $T(x)$.
- M: semantic rules, which map terms in $T(x)$ to fuzzy sets in U.

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3. TOPSIS and VIKOR in nonfuzzy environment

In this section, we present the TOPSIS and VIKOR methods in nonfuzzy environment. First, we provide the steps of each method. Then, we discuss variants of the methods used in various steps of these methods.

3.1. TOPSIS

The TOPSIS (Technique of Order Preference Similarity to the Ideal Solution) method [33, 87] is one of the most classical and widely-used MADM methods. The TOPSIS method is based in finding ideal and anti-ideal solutions and comparing the distance of each one of the alternatives to those. It has been successfully applied in various application areas, like supply chain management and logistics, engineering, marketing, and environmental management (for a review, see [5]).

The TOPSIS method is comprised of the following five steps:

- **Step 1. Calculation of the weighted normalized decision matrix**: The first step is to normalize the decision matrix in order to eliminate the units of the criteria. The normalized decision matrix is computed using the vector normalization technique as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{8}$$

  Then, the normalized decision matrix is multiplied with the weight associated with each of the criteria. The normalized weighted decision matrix is calculated as follows:

$$v_{ij} = w_j r_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{9}$$

  where w_j is the weight of the jth criterion.

- **Step 2. Determination of the ideal and anti-ideal solutions**: The ideal (A^+) and anti-ideal (A^-) solutions are computed as follows:

$$\begin{aligned} A^+ &= \left(v_1^+, v_2^+, \cdots, v_n^+\right) \\ &= \left\{ \left(\max_j v_{ij} | j \in \Omega_b \right), \left(\min_j v_{ij} | j \in \Omega_c \right) \right\}, j = 1, 2, \cdots, n \end{aligned} \tag{10}$$

$$\begin{aligned} A^- &= \left(v_1^-, v_2^-, \cdots, v_n^-\right) \\ &= \left\{ \left(\min_j v_{ij} | j \in \Omega_b \right), \left(\max_j v_{ij} | j \in \Omega_c \right) \right\}, j = 1, 2, \cdots, n \end{aligned} \tag{11}$$

  where Ω_b is the set of the benefit criteria and Ω_c is the set of the cost criteria.

- **Step 3. Calculation of the distance from the ideal and anti-ideal solutions**: The distance from the ideal and the anti-ideal solutions is computed for each alternative as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, i = 1, 2, \cdots, m \tag{12}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, i = 1, 2, \cdots, m \tag{13}$$

- **Step 4. Calculation of the relative closeness to the ideal solution**: The relative closeness of each alternative to the ideal solution is calculated as follows:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1, 2, \cdots, m \tag{14}$$

where $0 \leq C_i \leq 1$.

- **Step 5. Ranking the alternatives**: The alternatives are ranked from best (higher relative closeness value C_i) to worst.

The initial methodology was versatile enough to allow for various experiments and modifications; research has focused on the normalization procedure [8, 40, 54, 81, 93], the proper determination of the ideal and the anti-ideal solution [20, 30], and the metric used for the calculation of the distances from the ideal and the anti-ideal solution [12, 56, 64].

3.1.1. Normalization

A normalization method is applied in MADM techniques in order to convert the elements of the decision matrix into non-dimensional form. Hence, it is an important step in most MADM techniques. Although all normalization techniques have the same goal, i.e., scale the elements of the decision matrix to be approximately of the same magnitude, different normalization techniques may produce different solutions [13]. Thus, a normalization procedure may cause deviation from the originally recommended solutions and the best solution may be overlooked [13].

Jahan & Edwards [34] reviewed thirty one normalization techniques. They investigated how different normalization techniques can affect the decision making process in engineering design and showed that although many normalization methods are minor variants of each other, these nuances can have important consequences in decision making. Pavličić [66] studied the effect of simple, linear, and vector normalization on ELECTRE, TOPSIS, and SAW. He showed that the normalization technique affected the final decision. Zavadskas et al. [92] compared four linear normalization techniques with a nonlinear one proposed by Peldschus et al. [69] and showed that the nonlinear normalization technique improves the quality of the decision making. Milani et al. [54] studied the effect of five normalization techniques on the TOPSIS method and showed that the linear normalization techniques produce different closeness coefficients but the ranking of the alternatives remains the same, while the nonlinear normalization techniques generated a different ranking. Zavadskas et al. [93] evaluated the accuracy of nonlinear vector and linear normalization techniques and concluded that the relative closeness of the alternatives to the ideal solution is approximately 2.3 times less accurate in linear than in vector normalization. Migilinskas & Ustinovichius [53] studied eight normalization techniques and concluded that the normalization method must be chosen according to the objectives in order to meet special requirements. Peldschus [68] evaluated several normalization techniques and showed that a normalization technique affects the final ranking. In addition, Peldschus [68] showed that linear normalization does not ensure the stability of the solution. Chakraborty & Yeh [10] examined four normalization techniques on SAW. They concluded that the vector and max linear normalization techniques outperform other normalization techniques. Chakraborty & Yeh [11] extended their work on the TOPSIS method and concluded that vector normalization is more suitable for the TOPSIS method. Zavadskas & Turskis [91] proposed a new logarithmic normalization technique and compared it with two nonlinear normalization techniques. They concluded that the logarithmic normalization technique generated more stable solutions and can be used in problems where the elements of the decision matrix differ considerably. Çelen [8] examined the impact of four normalization techniques on the TOPSIS method and concluded that vector normalization is more suitable for the TOPSIS method. Vafaei et al. [81] evaluated the effect of six normalization techniques on the TOPSIS method and showed that the vector normalization technique is the best for the TOPSIS method, while the logarithmic normalization technique is the worst one.

In summary, many computational studies have shown that the vector normalization technique is more suitable for the TOPSIS method. However, the vector normalization technique is not the best technique for all case studies and it is important to select the appropriate normalization technique according to the case study's objectives. Hence, it is important for a decision maker to have access to different normalization techniques when applying the TOPSIS method. TOPSIS can be extended by using several normalization techniques. In the proposed DSS, we have incorporated the following normalization techniques:

1. Vector normalization

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{15}$$

   for benefit criteria, and

$$r_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{16}$$

   for cost criteria.

2. Linear sum normalization

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{17}$$

   for benefit criteria, and

$$r_{ij} = \frac{1/x_{ij}}{\sum_{i=1}^{m} 1/x_{ij}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{18}$$

   for cost criteria.

3. Linear max normalization

$$r_{ij} = \frac{x_{ij}}{x_j^+}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^+ = \max_i x_{ij} \tag{19}$$

   for benefit criteria, and

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^+}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^+ = \max_i x_{ij} \tag{20}$$

   for cost criteria.

4. Linear max-min normalization

$$r_{ij} = \frac{x_{ij} - x_j^-}{x_j^+ - x_j^-}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, \qquad (21)$$
$$x_j^+ = \max_i x_{ij}, x_j^- = \min_i x_{ij}$$

for benefit criteria, and

$$r_{ij} = \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, \qquad (22)$$
$$x_j^+ = \max_i x_{ij}, x_j^- = \min_i x_{ij}$$

for cost criteria.

5. Logarithmic normalization [91]

$$r_{ij} = \frac{\ln(x_{ij})}{\ln\left(\prod_{i=1}^{m} x_{ij}\right)}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \qquad (23)$$

for benefit criteria, and

$$r_{ij} = \frac{1 - \frac{\ln(x_{ij})}{\ln\left(\prod_{i=1}^{m} x_{ij}\right)}}{m - 1}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \qquad (24)$$

for cost criteria.

6. Marković method [51]

$$r_{ij} = 1 - \frac{x_{ij} - x_j^-}{x_j^+}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, \qquad (25)$$
$$x_j^+ = \max_i x_{ij}, x_j^- = \min_i x_{ij}$$

for both benefit and cost criteria.

7. Tzeng and Huang method [80]

$$r_{ij} = \frac{x_j^+}{x_{ij}}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^+ = \max_i x_{ij} \qquad (26)$$

for both benefit and cost criteria.

8. Nonlinear normalization [69]

$$r_{ij} = \left(\frac{x_{ij}}{x_j^+}\right)^2, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^+ = \max_i x_{ij} \qquad (27)$$

for benefit criteria, and

$$r_{ij} = \left(\frac{x_j^-}{x_{ij}}\right)^2, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^- = \min_i x_{ij} \tag{28}$$

for cost criteria.

9. Lai and Hwang method [41]

$$\begin{aligned} r_{ij} &= \frac{x_{ij}}{x_j^+ - x_j^-}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, \\ x_j^+ &= \max_i x_{ij}, x_j^- = \min_i x_{ij} \end{aligned} \tag{29}$$

for benefit criteria, and

$$\begin{aligned} r_{ij} &= \frac{x_{ij}}{x_j^- - x_j^+}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, \\ x_j^+ &= \max_i x_{ij}, x_j^- = \min_i x_{ij} \end{aligned} \tag{30}$$

for cost criteria.

10. Zavadskas and Turskis method [91]

$$r_{ij} = 1 - \left|\frac{x_j^+ - x_{ij}}{x_j^+}\right|, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^+ = \max_i x_{ij} \tag{31}$$

for benefit criteria, and

$$r_{ij} = 1 - \left|\frac{x_j^- - x_{ij}}{x_j^-}\right|, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n, x_j^- = \min_i x_{ij} \tag{32}$$

for cost criteria.

3.1.2. Ideal and anti-ideal solutions

The ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the anti-ideal solution minimizes the benefit criteria and maximizes the cost criteria. The simplest case to determine these solutions is that the ideal and anti-ideal points are fixed by the decision maker, but this should be avoided as it would mean that the decision maker could define a fixed ideal solution [59]. The determination of the ideal and anti-ideal solutions may affect the final ranking.

In the proposed DSS, we have incorporated the following methods to determine the ideal (A^+) and anti-ideal (A^-) solutions:

1. Max-min values

$$\begin{aligned} A^{+} &= \left(v_1^{+}, v_2^{+}, \cdots, v_n^{+}\right) \\ &= \left\{\left(\max_j v_{ij} | j \in \Omega_b\right), \left(\min_j v_{ij} | j \in \Omega_c\right)\right\}, j = 1, 2, \cdots, n \end{aligned} \tag{33}$$

$$\begin{aligned} A^{-} &= \left(v_1^{-}, v_2^{-}, \cdots, v_n^{-}\right) \\ &= \left\{\left(\min_j v_{ij} | j \in \Omega_b\right), \left(\max_j v_{ij} | j \in \Omega_c\right)\right\}, j = 1, 2, \cdots, n \end{aligned} \tag{34}$$

   where Ω_b is the set of the benefit criteria and Ω_c is the set of the cost criteria.

2. Absolute values

$$A^{+} = (1, 1, \cdots, 1) \tag{35}$$

$$A^{-} = (0, 0, \cdots, 0) \tag{36}$$

3. Fixed values

$$A^{+} = \left(\max_1, \max_2, \cdots, \max_n\right) \tag{37}$$

$$A^{-} = \left(\min_1, \min_2, \cdots, \min_n\right) \tag{38}$$

   where $\max_j$ and $\min_j$, $j = 1, 2, \cdots, n$, are the ideal and anti-ideal solutions for each criterion defined by the decision maker.

If the decision maker does not have any specific domain knowledge of the case study, he/she can select the ideal and anti-ideal solutions using the max-min or the absolute values of the criteria. On the other hand, domain knowledge of the case study can lead the decision maker to select fixed values.

3.1.3. Distance metrics

The distance from the ideal and the anti-ideal solutions can be computed using several distance metrics. In most cases, decision makers use Euclidean, Manhattan, or Chebyshev distance. Olson [56] utilized the Manhattan, Euclidean, and Chebyshev distance metrics in TOPSIS and concluded that the

Manhattan and Euclidean distance metrics seem very good for TOPSIS. Shih et al. [77] considered the Manhattan, Euclidean, Chebyshev, and weighted L_p distance metrics and also found that the Manhattan and Euclidean distance metrics are more consistent. Therefore, the Manhattan and Euclidean distance metrics seems to be the most appropriate distance measures for TOPSIS. However, the aforementioned studies did not take into account a large number of distance metrics that can be utilized in TOPSIS. Therefore, it is important for a decision maker to have access to different distance metrics when applying the TOPSIS method. The TOPSIS method can be extended by using several distance metrics. In the proposed DSS, we have incorporated the following distance metrics:

1. Manhattan distance

$$D_i^+ = \sum_{j=1}^{n} \left| v_{ij} - v_j^+ \right|, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{39}$$

$$D_i^- = \sum_{j=1}^{n} \left| v_{ij} - v_j^- \right|, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{40}$$

2. Euclidean distance

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{41}$$

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{42}$$

3. Chebyshev distance

$$D_i^+ = \max \left(\left| v_{ij} - v_j^+ \right| \right), i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{43}$$

$$D_i^- = \max \left(\left| v_{ij} - v_j^- \right| \right), i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{44}$$

4. Squared Euclidean distance

$$D_i^+ = \sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{45}$$

$$D_i^- = \sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{46}$$

5. Sørensen [78] or Bray-Curtis distance [7]

$$D_i^+ = \frac{\sum_{j=1}^{n} \left|v_{ij} - v_j^+\right|}{\sum_{j=1}^{n} \left(v_{ij} + v_j^+\right)}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{47}$$

$$D_i^- = \frac{\sum_{j=1}^{n} \left|v_{ij} - v_j^-\right|}{\sum_{j=1}^{n} \left(v_{ij} + v_j^-\right)}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{48}$$

6. Canberra distance [42]

$$D_i^+ = \sum_{j=1}^{n} \frac{\left|v_{ij} - v_j^+\right|}{v_{ij} + v_j^+}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{49}$$

$$D_i^- = \sum_{j=1}^{n} \frac{\left|v_{ij} - v_j^-\right|}{v_{ij} + v_j^-}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{50}$$

7. Lorentzian distance [24]

$$D_i^+ = \sum_{j=1}^{n} \ln(1 + \left|v_{ij} - v_j^+\right|), i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{51}$$

$$D_i^- = \sum_{j=1}^{n} \ln(1 + \left|v_{ij} - v_j^-\right|), i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{52}$$

8. Jaccard distance [65]

$$\begin{aligned} D_i^+ &= \frac{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}{\sum_{j=1}^{n} v_{ij}^2 + \sum_{j=1}^{n} v_j^{+2} - \sum_{j=1}^{n} v_{ij} v_j^+}, \\ &\quad i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \end{aligned} \tag{53}$$

$$\begin{aligned} D_i^+ &= \frac{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}{\sum_{j=1}^{n} v_{ij}^2 + \sum_{j=1}^{n} v_j^{-2} - \sum_{j=1}^{n} v_{ij} v_j^-}, \\ &\quad i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \end{aligned} \tag{54}$$

9. Dice distance [26]

$$D_i^+ = \frac{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}{\sum_{j=1}^{n} v_{ij}^2 + \sum_{j=1}^{n} v_j^{+2}}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{55}$$

$$D_i^- = \frac{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}{\sum_{j=1}^{n} v_{ij}^2 + \sum_{j=1}^{n} v_j^{-2}}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{56}$$

10. Bhattacharyya distance [6]

$$D_i^+ = -\ln\left(\sum_{j=1}^{n}\sqrt{v_{ij}v_j^+}\right)^2, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{57}$$

$$D_i^- = -\ln\left(\sum_{j=1}^{n}\sqrt{v_{ij}v_j^-}\right)^2, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{58}$$

11. Hellinger distance [24]

$$D_i^+ = 2\sqrt{1-\sum_{j=1}^{n}\sqrt{v_{ij}v_j^+}}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{59}$$

$$D_i^- = 2\sqrt{1-\sum_{j=1}^{n}\sqrt{v_{ij}v_j^-}}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{60}$$

12. Matusita distance [52]

$$D_i^+ = \sqrt{2-2\sum_{j=1}^{n}\sqrt{v_{ij}v_j^+}}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{61}$$

$$D_i^- = \sqrt{2-2\sum_{j=1}^{n}\sqrt{v_{ij}v_j^-}}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{62}$$

13. Squared-chord distance [72]

$$D_i^+ = \sum_{j=1}^{n}\left(\sqrt{v_{ij}}-\sqrt{v_j^+}\right)^2, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{63}$$

$$D_i^- = \sum_{j=1}^{n}\left(\sqrt{v_{ij}}-\sqrt{v_j^-}\right)^2, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{64}$$

14. Pearson χ^2 distance [67]

$$D_i^+ = \sum_{j=1}^{n}\frac{\left(v_{ij}-v_j^+\right)^2}{v_j^+}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{65}$$

$$D_i^- = \sum_{j=1}^{n}\frac{\left(v_{ij}-v_j^-\right)^2}{v_j^-}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{66}$$

15. Squared χ^2 distance [67]

$$D_i^+ = \sum_{j=1}^{n} \frac{\left(v_{ij} - v_j^+\right)^2}{v_{ij} + v_j^+}, i = 1, 2, \cdots, m, v_j^+ = \max_i v_{ij} \tag{67}$$

$$D_i^- = \sum_{j=1}^{n} \frac{\left(v_{ij} - v_j^-\right)^2}{v_{ij} + v_j^-}, i = 1, 2, \cdots, m, v_j^- = \min_i v_{ij} \tag{68}$$

3.2. VIKOR

The VIKOR (the acronym is in Serbian: VlseKriterijumska Optimizacija I Kompromisno Resenje, meaning multicriteria optimization and compromise solution) method has been developed to provide compromise solutions to discrete MADM problems that include non-commensurable and conflicting criteria. It has attracted much attention among researchers and has been applied in various areas, like design and manufacturing, supply chain management and logistics, heath care, and tourism management (for a review, see [85]).

The VIKOR method is comprised of the following five steps:

- **Step 1. Calculation of the aspired and tolerable levels**: The first step is to determine the best f_j^+ values (aspired levels) and the worst f_j^- values (tolerable levels) of all criterion functions, $j = 1, 2, \cdots, n$:

  $$f_j^+ = max_i f_{ij}, f_j^- = min_i f_{ij}, j = 1, 2, \cdots, n \tag{69}$$

  for benefit criteria, and

  $$f_j^+ = min_i f_{ij}, f_j^- = max_i f_{ij}, j = 1, 2, \cdots, n \tag{70}$$

  for cost criteria.

- **Step 2. Determination of the utility and the regret measures**: The utility measure S_i and the regret measure R_i are computed as follows:

  $$S_i = \sum_{j=1}^{n} w_j (f_j^+ - f_{ij})/(f_j^+ - f_j^-), i = 1, 2, \cdots, m \tag{71}$$

  $$R_i = max_j \left\{ w_j (f_j^+ - f_{ij})/(f_j^+ - f_j^-) \right\}, i = 1, 2, \cdots, m \tag{72}$$

- **Step 3. Calculation of the VIKOR index**: The VIKOR index is computed for each alternative as follows:

$$Q_i = v\left(S_i - S^+\right) / \left(S^- - S^+\right) + (1-v)\left(R_i - R^+\right) / \left(R^- - R^+\right), \quad i = 1, 2, \cdots, m \tag{73}$$

  where $S^+ = min_i S_i$, $S^- = max_i S_i$, $R^+ = min_i R_i$, $R^- = max_i R_i$; and v is the weight of the strategy of the maximum group utility (and is usually set to 0.5), whereas $1 - v$ is the weight of the individual regret.

- **Step 4. Ranking the alternatives**: The alternatives are ranked decreasingly by the values S_i, R_i and Q_i. The results are three ranking lists.

- **Step 5. Finding a compromise solution**: The alternative A^1, which is the best ranked by the measure Q (minimum), is proposed as a compromise solution if the following two conditions are satisfied:

  - **C1**. Acceptable advantage:

$$Q\left(A^2\right) - Q\left(A^1\right) \geq DQ \tag{74}$$

    where A^2 is the second best ranked by the measure Q and $DQ = \frac{1}{m-1}$; m is the number of alternative solutions.

  - **C2**. Acceptable stability in decision making: The alternative A^1 must also be the best ranked by the measures S or/and R. This compromise solution is stable within a decision making process, which could be one of the following strategies: (i) maximum group utility ($v > 0.5$), (ii) consensus ($v \approx 0.5$), or (iii) veto ($v < 0.5$).

  If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

  - Alternatives A^1 and A^2 if only condition $C2$ is not satisfied.

– Alternatives $A^1, A^2, \cdots, A^k$ if condition $C1$ is not satisfied; A^k is determined by the relation $Q\left(A^k\right) - Q\left(A^1\right) < DQ$ for maximum k (the positions of these alternative solutions are "in closeness").

These are the steps of the original version of the VIKOR method that is used in the implemented decision support system. The method was extended at a later stage with 4 new steps which provided a stability analysis to determine the weight stability intervals and included a trade-off analysis [60, 61]. The VIKOR method has been modified to handle alternatives with different criteria, uncertainty, etc., but the steps of the method have not been modified. Contrary to the many alternative methodologies that can be used in various steps of TOPSIS, there are only two modifications that can be made in the steps of VIKOR. First of all, we can experiment with the selection of the ideal and anti-ideal solutions. As already described in subsection 3.1.2, there are three different methods than can be used to determine the ideal (A^+) and anti-ideal (A^-) solutions:

- Max-min values (Equations (33) and (34))
- Absolute values (Equations (35) and (36) and using either the max ranking or a fixed value for the maximum)
- Fixed values (Equations (37) and (38))

In addition, decision makers can select the weight of the maximum group utility strategy (v).

4. TOPSIS and VIKOR in fuzzy environment for group decision making

4.1. Fuzzy TOPSIS

The TOPSIS method was further extended to handle fuzzy numbers involving the opinions of a number of independent experts. There are many fuzzy

TOPSIS extensions [14, 23, 46, 83]. Table 1, adopted from [37], presents a comparison of the various fuzzy TOPSIS methods proposed in the literature. In this Section, we present a fuzzy extension of TOPSIS that is based on Chen's methodology [14] and uses triangular fuzzy numbers. Then, we discuss variations that can be used in this method, focusing on the distance measurement, the determination of the ideal and anti-ideal points, and the use of other than triangular fuzzy numbers, like trapezoidal fuzzy numbers.

In conjunction with the steps of the typical TOPSIS method presented earlier in Section 3.1, the steps of the fuzzy extension are:

- **Step 1. Identification of the evaluation criteria**: If we assume that the decision group has K persons, then the importance of the criteria and the ratings of the alternatives can be calculated as:

$$\widetilde{x}_{ij} = \frac{1}{K}\left[\widetilde{x}_{ij}^{1}(+)\widetilde{x}_{ij}^{2}(+)\cdots(+)\widetilde{x}_{ij}^{K}\right] \tag{75}$$

$$\widetilde{w}_{j} = \frac{1}{K}\left[\widetilde{w}_{j}^{1}(+)\widetilde{w}_{j}^{2}(+)\cdots(+)\widetilde{w}_{j}^{K}\right] \tag{76}$$

  where $\widetilde{x}_{ij}^{K}$ and $\widetilde{w}_{j}^{K}$ are the ratings and criteria weights of the Kth decision maker.

- **Step 2. Selection of the linguistic variables**: Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to the criteria.

- **Step 3. Aggregations**: Aggregate the weight of criteria to get the aggregated fuzzy weight $\widetilde{w}_j$ of criterion C_j, and pool the decision makers' opinions to get the aggregated fuzzy rating $\widetilde{x}_{ij}$ of alternative A_i under criterion C_j.

- **Step 4. Construction of the fuzzy decision matrix and the normalized fuzzy decision matrix**: The fuzzy decision matrix is con-

Table 1: A comparison of fuzzy TOPSIS methods [37]

Source	Attribute weights	Type of fuzzy numbers	Ranking method	Normalization method
Chen and Hwang [18]	Fuzzy numbers	Trapezoidal	Lee and Li's [43] generalized mean method	Linear normalization
Liang [46]	Fuzzy numbers	Trapezoidal	Chen's [16] ranking with maximizing set and minimizing set	Linear normalization
Chen [14]	Fuzzy numbers	Triangular	Chen [14] proposes the vertex method	Linear normalization
Chu [22]	Fuzzy numbers	Triangular	Liou and Wang's [48] ranking method of total integral value with $\alpha = 1/2$	Modified manhattan distance
Tsaur et al. [79]	Crisp values	Triangular	Zhao and Govind's [95] center of area method	Vector normalization
Zhang and Lu [94]	Crisp values	Triangular	Chen's [14] vertex mode	Manhattan distance
Chu and Lin [23]	Fuzzy numbers	Triangular	Kaufmann and Gupta's [kau] mean of the removals method	Linear normalization
Cha and Yung [9]	Crisp values	Triangular	Cha and Young [9] propose a fuzzy distance operator	Linear normalization
Yang and Hung [84]	Fuzzy numbers	Triangular	Chen's [14] vertex method	Normalized fuzzy linguistic ratings are used
Wang and Elhag [83]	Fuzzy numbers	Triangular	Chen's [14] vertex method	Linear normalization
Jahanshahloo et al. [35]	Crisp values	Interval data	Jahanshahloo et al. [35] propose a new column and ranking method	

structed as:

$$\widetilde{D}=\begin{bmatrix}\widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n}\\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn}\end{bmatrix} \tag{77}$$

and the vector of the criteria weights as:

$$\widetilde{W}=[\widetilde{w}_1,\widetilde{w}_2,\cdots,\widetilde{w}_n] \tag{78}$$

where $\widetilde{x}_{ij}$ and $\widetilde{w}_j, i=1,2,\cdots,m, j=1,2,\cdots,n$ are linguistic variables according to Step 2. They can be described by the triangular fuzzy numbers $\widetilde{x}_{ij}=(a_{ij},b_{ij},c_{ij})$ and $\widetilde{w}_j=(w_{j1},w_{j2},w_{j3})$. For the normalization step, Chen uses the linear scale transformation in order to drop the units and make the criteria comparable; it is also important to preserve the property stating that the ranges of the normalized triangular fuzzy numbers belong to $[0,1]$. The normalized fuzzy decision matrix denoted by $\widetilde{R}$ is:

$$\widetilde{R}=[\widetilde{r}_{ij}]_{m\times n} \tag{79}$$

The set of benefit criteria is B and the set of cost criteria is C, therefore:

$$\widetilde{r}_{ij}=\left(\frac{a_{ij}}{c_j^+},\frac{b_{ij}}{c_j^+},\frac{c_{ij}}{c_j^+}\right), j\in B, i=1,2,\cdots,m \tag{80}$$

$$\widetilde{r}_{ij}=\left(\frac{a_j^-}{c_{ij}},\frac{a_j^-}{b_{ij}},\frac{a_j^-}{a_{ij}}\right), j\in C, i=1,2,\cdots,m \tag{81}$$

$$c_j^+=\max_i c_{ij},\ \text{if } j\in B, i=1,2,\cdots,m \tag{82}$$

$$a_j^-=\min_i a_{ij},\ \text{if } j\in C, i=1,2,\cdots,m \tag{83}$$

- **Step 5. Construction of the fuzzy weighted normalized decision matrix**: Then, the fuzzy weighted normalized decision matrix can be constructed as:

$$\widetilde{V}=[\widetilde{v}_{ij}]_{m\times n}, i=1,2,\cdots,m, j=1,2,\cdots,n \tag{84}$$

where:

$$\widetilde{v}_{ij} = \widetilde{r}_{ij}(\cdot)\widetilde{w}_j, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{85}$$

The elements $\widetilde{v}_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, are normalized positive triangular fuzzy numbers ranging from 0 to 1.

- **Step 6. Determination of the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS)**: The fuzzy positive ideal solution (FPIS, A^+) and the fuzzy negative ideal solution (FNIS, A^-) are:

$$A^+ = \left(\widetilde{v}_1^+, \widetilde{v}_2^+, \cdots, \widetilde{v}_n^+\right) \tag{86}$$

$$A^- = \left(\widetilde{v}_1^-, \widetilde{v}_2^-, \cdots, \widetilde{v}_n^-\right) \tag{87}$$

where:

$$v_j^+ = (1, 1, 1), \widetilde{v}_j^- = (0, 0, 0), j = 1, 2, \cdots, n \tag{88}$$

- **Step 7. Calculation of the distance of each alternative from FPIS and FNIS**: The distance of each of the alternatives from FPIS and FNIS can be calculated as:

$$D_i^+ = \sum_{j=1}^{n} d\left(\widetilde{v}_{ij}, \widetilde{v}_j^+\right), i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{89}$$

$$D_i^- = \sum_{j=1}^{n} d\left(\widetilde{v}_{ij}, \widetilde{v}_j^-\right), i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{90}$$

where d is the distance measurement between two fuzzy numbers.

- **Step 8. Calculation of the closeness coefficient of each alternative**: The closeness coefficient of each alternative can be defined as:

$$CC_i = \frac{d_i^-}{D_i^+ + D_i^-}, i = 1, 2, \cdots, m \tag{91}$$

- **Step 9. Ranking the alternatives**: An alternative A_i is better than A_j if its closeness coefficient is closer to 1. Therefore, the final ranking of the alternatives is defined by the value of the closeness coefficient.

As already mentioned, there are many other fuzzy TOPSIS extensions focusing on the distance measurement [9, 14, 50], the determination of the ideal and anti-ideal points [14, 50], and the use of other than triangular fuzzy numbers, like trapezoidal fuzzy numbers [46, 50]. The fuzzy TOPSIS method and the methods presented in subsections 4.1.1 and 4.1.2 can be modified using trapezoidal fuzzy numbers. In the proposed DSS, we allow the decision maker to select either triangular or trapezoidal fuzzy numbers to use in the fuzzy TOPSIS method.

4.1.1. Distance metrics

The distance from the ideal and the anti-ideal solutions can be computed using several distance metrics. In most fuzzy TOPSIS extensions, decision makers use Chen's vertex method [14]. An interesting generalization of the fuzzy TOPSIS method is proposed by Dymova et al. [27]. Instead of defuzzifying the initial fuzzy decision matrix and converting fuzzy values to real-values ones, they treat the distances of the alternatives from the ideal and anti-ideal solutions as modified weighted sums of local criteria. Therefore, they avoid using weighted sums and proposed the utilization of local criteria aggregation.

The selection of a distance metric may affect the final ranking. In general, most works select Chen's vertex method [14] as a distance metric for fuzzy TOPSIS. However, the fuzzy TOPSIS method can be extended by using several distance measures of fuzzy numbers. In the following distance metrics, we want to calculate the distance between two triangular fuzzy numbers $\widetilde{m} = (m_1, m_2, m_3)$ and $\widetilde{n} = (n_1, n_2, n_3)$. In the proposed DSS, we have incorporated the following distance metrics:

1. Chen's vertex method [14]

$$d(\widetilde{m}, \widetilde{n}) = \sqrt{\frac{1}{3}\left[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2\right]} \tag{92}$$

2. The fuzzy distance operator proposed by Cha & Jung [9]

$$\begin{aligned} d(\widetilde{m}, \widetilde{n}) = &\left|\frac{m_1 - n_1 + m_2 - n_2 + m_3 - n_3}{3}\right| \\ &+ \left|\frac{m_1 + m_3 - 2m_2}{2\,(m_3 - m_1)} - \frac{n_1 + n_3 - 2n_2}{2\,(n_3 - n_1)}\right| \times \left|\frac{m_3 - m_1 + n_3 - n_1}{4}\right| \end{aligned} \tag{93}$$

3. The fuzzy distance measure proposed by Lin [47]

$$d(\widetilde{m},\widetilde{n}) = \sqrt{\frac{1}{6}\left[\sum_{i=1}^{3}(n_i - m_i)^2 + (n_2 - m_2)^2 + \sum_{i=1}^{2}(n_i - m_i)(n_{i+1} - m_{i+1})\right]} \tag{94}$$

4. A distance measure based on the similarity measure proposed by Chen [19]

$$d(\widetilde{m},\widetilde{n}) = \frac{\sum_{i=1}^{3}|m_i - n_i|}{3} \tag{95}$$

5. A distance measure based on the similarity measure proposed by Chen & Hsieh [17]

$$d(\widetilde{m},\widetilde{n}) = \left|\frac{m_1 + 4m_2 + m_3}{6} - \frac{n_1 + 4n_2 + n_3}{6}\right| \tag{96}$$

4.1.2. Ideal and anti-ideal solutions

The simplest case to determine these solutions is that the ideal and anti-ideal points are fixed by the decision maker, as in the fuzzy TOPSIS variant presented in Section 4.1. The determination of the ideal and anti-ideal solutions may affect the final ranking.

In the proposed DSS, we have incorporated the following methods to determine the ideal ($A^+ = \left(\widetilde{v}_1^+, \widetilde{v}_2^+, \cdots, \widetilde{v}_n^+\right)$) and anti-ideal ($A^- = \left(\widetilde{v}_1^-, \widetilde{v}_2^-, \cdots, \widetilde{v}_n^-\right)$) solutions:

1. Max-min values

$$\widetilde{v}_j^+ = \max_i \widetilde{v}_{ij}, \widetilde{v}_j^- = \min_i \widetilde{v}_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{97}$$

2. Absolute values

$$\widetilde{v}_j^+ = (1, 1, 1), \widetilde{v}_j^- = (0, 0, 0), j = 1, 2, \cdots, n \tag{98}$$

3. Fixed values

$$\widetilde{v}_j^+ = \left(\max_1, \max_2, \max_3\right), \widetilde{v}_j^- = \left(\min_1, \min_2, \min_3\right), j = 1, 2, \cdots, n \tag{99}$$

where $\max_k$ and $\min_k$, $k = 1, 2, 3$, are the ideal and anti-ideal solutions for each criterion defined by the decision makers using a fuzzy triangular number.

4.2. Fuzzy VIKOR

Similar to the TOPSIS method, the VIKOR method was further extended to handle fuzzy numbers involving the opinions of a number of independent experts. There are many fuzzy VIKOR extensions [57, 76, 82, 88]. Table 2 presents a comparison of the various fuzzy VIKOR methods proposed in the literature. In this Section, we present a fuzzy extension of VIKOR that is based on the methodology proposed by Sanayei et al. [75] and uses trapezoidal fuzzy numbers. We will present this method using triangular fuzzy numbers (as we did in Section 4.1 for the fuzzy TOPSIS method). Then, we discuss variations that can be used in this method, focusing on the defuzzification technique and the use of other than triangular fuzzy numbers, like trapezoidal fuzzy numbers.

The steps of the fuzzy VIKOR method are:

- **Step 1. Identification of the problem objectives and scope**: The decision goals and the scope of the problem are defined. Then, the objectives of the decision making process are identified.

- **Step 2. Identification of the criteria**: We form a group of decision makers to identify the criteria and their evaluation scales.

- **Step 3. Identification of the appropriate linguistic variables**: Choose the appropriate linguistic variables for the importance weights of the criteria and the linguistic ratings for the alternatives with respect to the criteria.

- **Step 4. Calculation of the aggregated fuzzy weight of criteria and the aggregated fuzzy rating of alternatives**: Let the fuzzy rating and importance weight of the kth decision maker be $\widetilde{x}_{ijk} = (\widetilde{x}_{ijk1}, \widetilde{x}_{ijk2}, \widetilde{x}_{ijk3})$ and $\widetilde{w}_{ijk} = (\widetilde{w}_{ijk1}, \widetilde{w}_{ijk2}, \widetilde{w}_{ijk3})$, respectively, where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$. Hence, the aggregated fuzzy ratings $(\widetilde{x}_{ij})$ of alternatives with respect to each criterion can be calculated as:

$$\widetilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}) \tag{100}$$

Table 2: A comparison of fuzzy VIKOR methods

Source	Type of fuzzy numbers	Defuzzification method
Opricovic [57]	Triangular	2nd weighted mean
Rostamzadeh et al. [73]	Triangular	Center of gravity
Chen and Wang [15]	Triangular	Minimizing and maximizing sets
Wan et al. [82]	Triangular	Custom defuzzification method
Shemshadi et al. [76]	Trapezoidal	Custom defuzzification method
Ju and Wang [36]	Trapezoidal	Center of gravity
Yucenur and Demirel [88]	Trapezoidal	Center of gravity
Opricovic and Tzeng [58]	Trapezoidal	Custom defuzzification method
Liu et al. [49]	Trapezoidal	Center of gravity

where:

$$x_{ij1} = \min_k \{x_{ijk1}\}, x_{ij2} = \frac{1}{K}\sum_{k=1}^{K} x_{ijk2}, x_{ij3} = \max_k \{x_{ijk3}\} \tag{101}$$

The aggregated fuzzy weights ($\widetilde{w}_j$) of each criterion can be calculated as:

$$\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3}) \tag{102}$$

where:

$$w_{j1} = \min_k \{w_{jk1}\}, w_{j2} = \frac{1}{K}\sum_{k=1}^{K} w_{jk2}, w_{ij3} = \max_k \{w_{jk3}\} \tag{103}$$

The problem can be concisely expressed in matrix format as follows:

$$\widetilde{D} = \begin{bmatrix} \widetilde{x}_{11} & \widetilde{x}_{12} & \cdots & \widetilde{x}_{1n} \\ \widetilde{x}_{21} & \widetilde{x}_{22} & \cdots & \widetilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{x}_{m1} & \widetilde{x}_{m2} & \cdots & \widetilde{x}_{mn} \end{bmatrix} \tag{104}$$

and the vector of the criteria weights as:

$$\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \cdots, \widetilde{w}_n] \tag{105}$$

where $\widetilde{x}_{ij}$ and $\widetilde{w}_j, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n$, are linguistic variables according to Step 3. They can be approximated by the triangular fuzzy numbers $\widetilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ and $\widetilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$.

- **Step 5. Defuzzification of the fuzzy decision matrix and the fuzzy weight of each criterion into crisp values**: Defuzzify the fuzzy decision matrix and fuzzy weight of each criterion into crisp values using COG defuzzification relation (see Section 4.2.1).

- **Step 6. Calculation of the best and the worst values of all criteria functions**: Determine the best f_j^+ and the worst f_j^- values of all criteria functions:

$$f_j^+ = \max_i f_{ij}, f_j^- = \min_i f_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{106}$$

if the jth function is to be maximized (benefit) and:

$$f_j^+ = \min_i f_{ij}, f_j^- = \max_i f_{ij}, i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \tag{107}$$

if the jth function is to be minimized (cost).

- **Step 7. Computation of the values S_i and R_i**: Compute the values S_i and R_i using the relations:

$$\begin{gathered} S_i = \sum_{j=1}^{n} w_j \left(f_j^+ - f_{ij}\right) / \left(f_j^+ - f_j^-\right), \\ i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \end{gathered} \tag{108}$$

$$\begin{gathered} R_i = \max_j \left[w_j \left(f_j^+ - f_{ij}\right) / \left(f_j^+ - f_j^-\right)\right], \\ i = 1, 2, \cdots, m, j = 1, 2, \cdots, n \end{gathered} \tag{109}$$

- **Step 8. Computation of the values Q_i**: Compute the values Q_i using the relation:

$$\begin{gathered} Q_i = v\left(S_i - S^+\right) / \left(S^- - S^+\right) + (1 - v)\left(R_i - R^+\right) / \left(R^- - R^+\right), \\ i = 1, 2, \cdots, m \end{gathered} \tag{110}$$

where $S^+ = \min_i S_i$; $S^- = \max_i S_i$; $R^+ = \min_i R_i$; $R^- = \max_i R_i$; and v is introduced as a weight for the strategy of the ”maximum group utility”, whereas $1 - v$ is the weight of the individual regret.

- **Step 9. Ranking the alternatives**: Rank the alternatives, sorting by the values S, R, and Q in ascending order. The results are three ranking lists.

- **Step 10. Proposal of a compromise solution**: Propose as a compromise solution the alternative $[A^{(1)}]$, which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied:

– C1 - Acceptable advantage

$$Q\left(A^{(2)}\right) - Q\left(A^{(1)}\right) \geq DQ \tag{111}$$

where $A^{(2)}$ is the second ranked alternative by the measure Q and $DQ = 1/(m-1)$.

– C2 - Acceptable stability in decision making: The alternative $A^{(1)}$ must also be the best ranked by S and/or R. This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility ($v > 0.5$), or "by consensus" ($v \approx 0.5$), or "with veto" ($v < 0.5$). If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

  * Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
  * Alternatives $A^{(1)}, A^{(2)}, \cdots, A^{(l)}$ if the condition C1 is not satisfied; $A^{(l)}$ is determined by the relation $Q(A^{(l)}) - Q(A^{(1)}) < DQ$ for maximum l (the positions of these alternatives are "in closeness").

As already mentioned, there are many other fuzzy VIKOR extensions focusing on the defuzzification technique [2, 57, 76] and the use of other than triangular fuzzy numbers, like trapezoidal fuzzy numbers [36, 75, 88]. The fuzzy VIKOR method and the methods presented in subsection 4.2.1 can be modified using trapezoidal fuzzy numbers. In the proposed DSS, we allow the decision maker to select either triangular or trapezoidal fuzzy numbers to use in the fuzzy TOPSIS method. Similar to the nonfuzzy VIKOR method, decision makers can select the weight of the maximum group utility strategy (v) and the method that will be used to determine the ideal (A^+) and anti-ideal (A^-) solutions:

• Max-min values (Equations (33) and (34))

- Absolute values (Equations (35) and (36) and using either the max ranking or a fixed value for the maximum)

- Fixed values (Equations (37) and (38))

4.2.1. Defuzzification

In all variants of the fuzzy VIKOR method, a defuzzification technique is necessary to convert fuzzy numbers to crisp values. There are many defuzzification techniques proposed in the literature (for a literature review, see [45]). The use of each one of these techniques can have a substantial impact on the output of the fuzzy VIKOR method. Leekwijck and Kerre concludes that the maxima methods behave well with respect to core selection, scale invariance, monotony and the triangular conorm criterion. The center of gravity method does not fulfill the basic defuzzification criteria but it provides the highly practical property of continuity. Most of the defuzzifications techniques presented below has not been incorporated into the fuzzy VIKOR method. Therefore, it is important for the decision maker to experiment with these techniques on specific case studies.

All defuzzification techniques can be formulated both in discrete and in continuous form. Without loss of generality and for simplicity, we will use the discrete formulation. In the proposed DSS, we have incorporated the following defuzzification metrics:

1. First of maxima (FOM): FOM method selects the smallest element of the core of A as the defuzzification value:

$$FOM(A) = \min\left(\text{core}(A)\right) \tag{112}$$

2. Last of maxima (LOM): LOM method selects the greatest element of the core of A as the defuzzification value:

$$LOM(A) = \max\left(\text{core}(A)\right) \tag{113}$$

3. Middle of maxima (MOM): If the core of A contains an odd number of elements, then the middle element of the core is selected such that:

$$\left|\text{core}(A)_{<MOM(A)}\right| = \left|\text{core}(A)_{>MOM(A)}\right| \tag{114}$$

If the core of A contains an even number of elements, then we can select an element as the defuzzification value such that:

$$\left|\text{core}(A)_{<MOM(A)}\right| = \left|\text{core}(A)_{>MOM(A)}\right| \pm 1 \tag{115}$$

4. Center of gravity (COG): COG method calculates the center of gravity of the area under the membership function:

$$\frac{\sum_{x_{min}}^{x_{max}} x\mu_A(x)}{\sum_{x_{min}}^{x_{max}} \mu_A(x)} \tag{116}$$

5. Mean of maxima (MeOM): MeOM method is a variant of COG method. It computes the mean of all the elements of the core of A:

$$MeOM(A) = \frac{\sum_{x \in \text{core}(A)} x}{|\text{core}(A)|} \tag{117}$$

6. Basic defuzzification distributions (BADD): BADD method [29] is an extension of the COG method. The defuzzification value is computed as follows:

$$BADD(A) = \frac{\sum_{x_{min}}^{x_{max}} x\mu_A^{\gamma}(x)}{\sum_{x_{min}}^{x_{max}} \mu_A^{\gamma}(x)} \tag{118}$$

where γ is a free parameter in $[0, \infty)$. The parameter γ is used to adjust the method to the following special cases:

$$\begin{cases} BADD(A) = MeOS(A), & \text{if } \gamma = 0 \\ BADD(A) = COG(A), & \text{if } \gamma = 1 \\ BADD(A) = MeOM(A), & \text{if } \gamma \to \infty \end{cases} \tag{119}$$

where $MeOS(A)$ is the mean of support of the core A.

7. Indexed center of gravity (ICOG): ICOG method [29] computes the center of gravity of the fuzzy set that is obtained after putting all membership values below a certain threshold α equal to zero:

$$ICOG(A, \alpha) = \frac{\sum_{x \in A_\alpha} x\mu_A(x)}{\sum_{x \in A_\alpha} \mu_A(x)} \tag{120}$$

In the proposed DSS, we have implemented ICOG using $\alpha = 0$, therefore all values are included in the calculation of the crisp value.

8. Bisector of area (BOA): The bisector is the vertical line that divides the region of the fuzzy number into two subregions of equal area:

$$\sum_{x_{min}}^{BOA} x\mu_A(x) = \sum_{BOA}^{x_{max}} x\mu_A(x) \tag{121}$$

5. Implementation and presentation of the Decision Support System

Desktop decision support systems require all involved decision makers to be at the same location or use different communication channels to collaborate. In most group decision making problems that involve various decision makers and stakeholders, all decision makers should evaluate the criteria and alternatives of the problem. That usually requires that each decision maker will fill out questionnaires and the answers will be later aggregated. In order to eliminate this problem, we implemented the proposed DSS as a web-based one. Web-based decision support systems have reduced technological barriers and made it easier and less costly to make decisions in a group decision making environment.

The web-based decision support system has been implemented using PHP, MySQL, Ajax, and jQuery. The DSS is implemented with a responsive web design, i.e., the web pages look equally good regardless of the screen size of a device. That allows decision makers perform all requested steps using their mobile device, tablet, or computer. Since the DSS can be used in a group decision making environment, it is important to allow decision makers use the DSS without any time or geographical constraints. Therefore, decision makers in geographically distributed locations can access the DSS and add their evaluations without directly interacting with each other.

Figure 2 presents the decision making process that the decision maker should follow. Initially, the decision maker selects the type of MADM methodologies that he/she wishes to use, i.e., use of traditional TOPSIS and VIKOR or fuzzy TOPSIS and VIKOR involving the opinions of a number of independent experts. In case that the decision maker selects to use the traditional TOPSIS and VIKOR methods, then he/she is asked to insert the alternatives and criteria of the problem (Figure 3). The decision maker has the option either to upload an Excel file with all the available information (based on an Excel template that can be downloaded from the DSS) or to insert manually the information. For each criterion, the decision maker enters the name, the type (qualititative or quantitative criterion), the type of optimization (min or max), and the weight of the criterion. For each alternative, the decision maker enters the name and the ranking associated with each criterion. Then, the decision maker selects the algorithms and parameters. In each scenario, the decision maker can select to run both TOPSIS and VIKOR and decide how many different methods wants to combine. As detailed in Section 3, the following methods are available (Figure 4):

- TOPSIS
  - Calculation of ideal and anti-ideal solutions
    - Max-min values (Equations (33) and (34))
    - Absolute values (Equations (35) and (36))
    - Fixed values (Equations (37) and (38))
  - Normalization method
    - Vector normalization (Equations (15) and (16))
    - Linear sum normalization (Equations (17) and (18))
    - Linear max normalization (Equations (19) and (20))
    - Linear max-min normalization (Equations (21) and (22))
    - Logarithmic normalization (Equations (23) and (24))
    - Marković method (Equation (25))
    - Tzeng and Huang method (Equation (26))
    - Nonlinear normalization (Equations (27) and (28))
    - Lai and Hwang method (Equations (29) and (30))
    - Zavadskas and Turskis method (Equations (31) and (32))
  - Distance metric

        * Manhattan distance (Equations (39) and (40))
        * Euclidean distance (Equations (41) and (42))
        * Chebyshev distance (Equations (43) and (44))
        * Squared Euclidean distance (Equations (45) and (46))
        * Sorensen or Bray-Curtis distance (Equations (47) and (48))
        * Canberra distance (Equations (49) and (50))
        * Lorentzian distance (Equations (51) and (52))
        * Jaccard distance (Equations (53) and (54))
        * Dice distance (Equations (55) and (56))
        * Bhattacharyya distance (Equations (57) and (58))
        * Hellinger distance (Equations (59) and (60))
        * Matusita distance (Equations (61) and (62))
        * Squared-chord distance (Equations (63) and (64))
        * Pearson χ^2 distance (Equations (65) and (66))
        * Squared χ^2 distance (Equations (67) and (68))

- VIKOR
    - Calculation of ideal and anti-ideal solutions
        * Max-min values (Equations (33) and (34))
        * Absolute values (Equations (35) and (36) and using either the max ranking or a fixed value for the maximum)
        * Fixed values (Equations (37) and (38))
    - the weight of the maximum group utility strategy (v)

After entering the data and selecting the suitable methods and parameters, the DSS displays the results both in a graphical and in a tabular format. The decision makers can export the results in pdf format for further processing. Figure 5 displays the results of a problem where the decision maker selected to use TOPSIS and VIKOR with various method combinations. The final results of all different combinations are shown. The closeness coefficient for TOPSIS variants and the measure Q for VIKOR variants is displayed (note that for VIKOR variants we use $1 - Q$ in the figure in order for the decision maker to be able to compare the results with the closeness coefficients of the TOPSIS variants; of course, tabular results contain all information for making a decision).

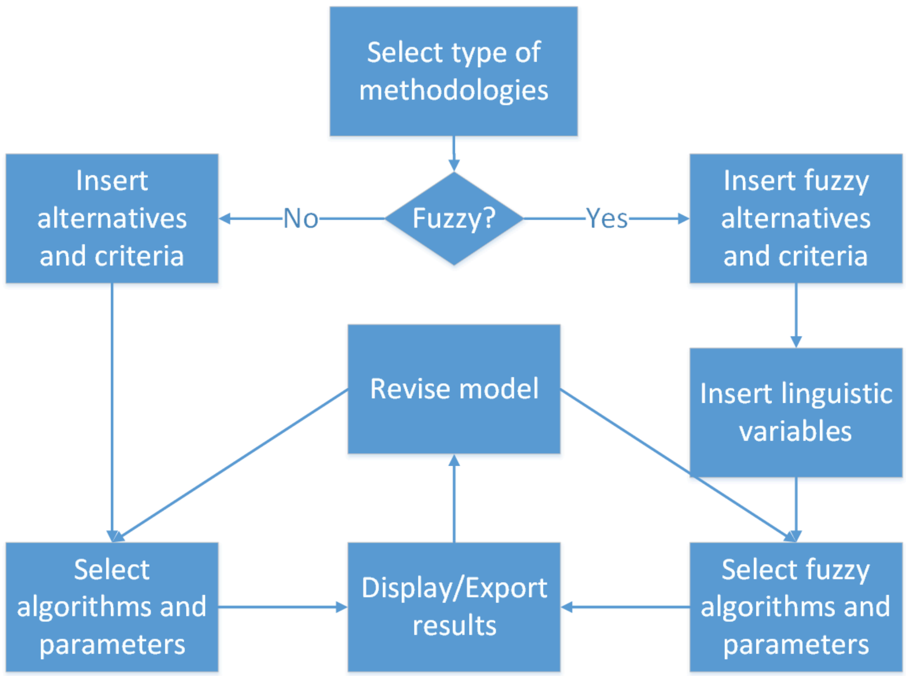


Figure 2: Decision making process

In case that the decision maker selects to use the fuzzy TOPSIS and VIKOR methods, we assume that we have already formed a group of decision makers and one of them acts as the leader of the group. Initially, the leader is asked to insert the alternatives and criteria of the problem (Figure 6). He/she should enter the name and type (benefit or cost) of each criterion and the name of each alternative. Next, the leader selects the type of fuzzy numbers (triangular or trapezoidal) and enters the linguistic variables for the alternatives and criteria (Figure 7). Then, the leader selects the algorithms and parameters. In each

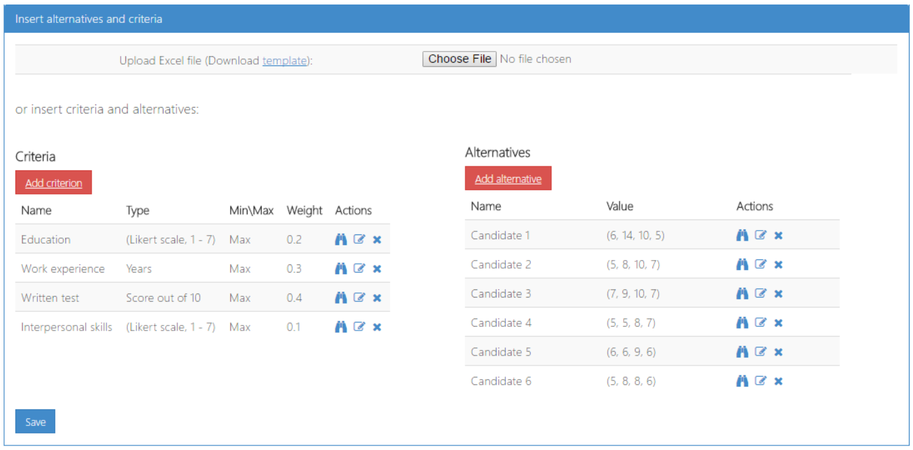


Figure 3: Insert alternatives and criteria

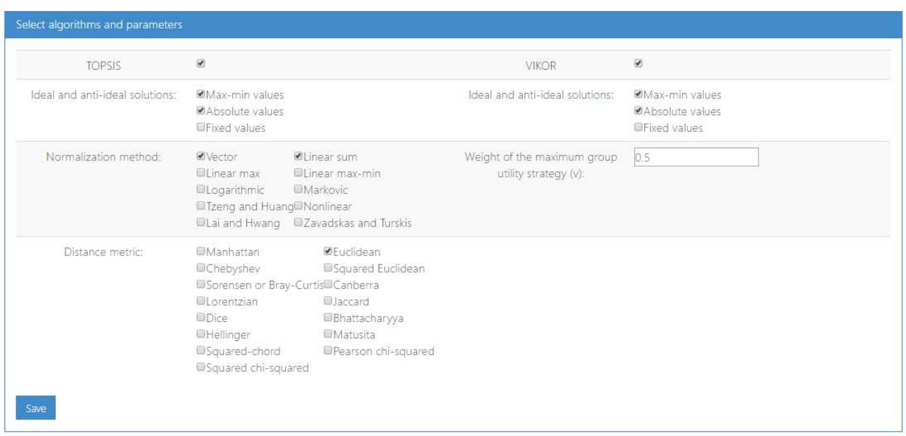
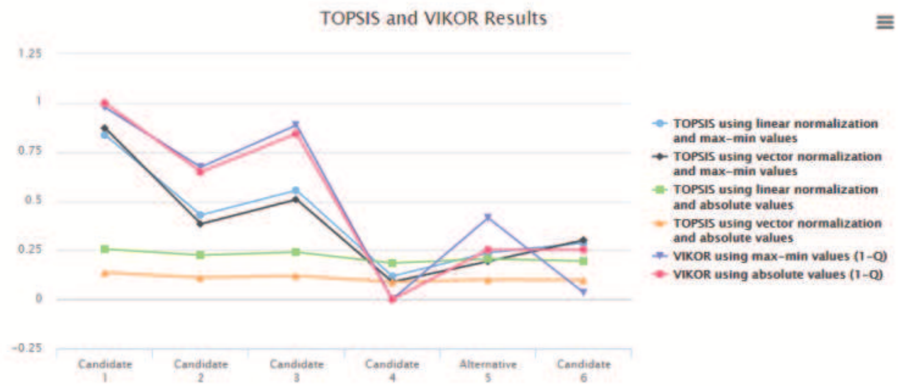


Figure 4: Select algorithms and parameters



	Alternatives						Ranking
	C1	C2	C3	C4	C5	C6	
TOPSIS using linear normalization and max-min values	0.837	0.430	0.555	0.118	0.237	0.287	C1, C3, C2, C6, C5, C4
TOPSIS using vector normalization and max-min values	0.872	0.384	0.508	0.089	0.193	0.302	C1, C3, C2, C6, C5, C4
TOPSIS using linear normalization and absolute values	0.256	0.225	0.240	0.184	0.206	0.194	C1, C3, C2, C5, C6, C4

Figure 5: Results of TOPSIS and VIKOR

scenario, he/she can select to run both fuzzy TOPSIS and VIKOR and decide how many different methods wants to combine. As detailed in Section 4, the following methods are available (Figure 8):

- Fuzzy TOPSIS
  - Calculation of ideal and anti-ideal solutions
    - Max-min values (Equations (33) and (34))
    - Absolute values (Equations (35) and (36))
    - Fixed values (Equations (37) and (38))
  - Distance metric
    - Chen's vertex method (Equation (92))
    - Cha & Jung method (Equation (93))
    - Lin's method (Equation (94))
    - Chen's distance metric (Equation (95))

        * Chen & Hsieh distance metric (Equation (96))

      .

- Fuzzy VIKOR
    - Calculation of ideal and anti-ideal solutions
        * Max-min values (Equations (33) and (34))
        * Absolute values (Equations (35) and (36) and using either the max ranking or a fixed value for the maximum)
        * Fixed values (Equations (37) and (38))
    - Defuzzification technique
        * First of maxima (Equation (112))
        * Last of maxima (Equation (113))
        * Middle of maxima (Equation (114))
        * Center of gravity (Equation (116))
        * Mean of maxima (Equation (117))
        * Mean of support (Equations (118) and (119))
        * ICOG (Equation (120))
        * Bisector of area (Equation (121))
    - the weight of the maximum group utility strategy (v)

In the next step, each decision maker evaluates the criteria and alternatives using a linguistic variable (Figure 9). When all decision makers have entered their evaluations, the leader can see the results of the fuzzy TOPSIS and/or VIKOR methods. The results are graphically (Figure 10) and numerically displayed (Figure 11). The DSS can also output a thorough report in a pdf file containing the results of the methods. Similar to the results of the traditional TOPSIS and VIKOR methods, the closeness coefficient for fuzzy TOPSIS variants and the measure Q for VIKOR variants is displayed. The decision makers can also revise the original model in order to accommodate any needed changes according to the feedback from the first run. In this way, they can fine-tune the model and find more appropriate solutions.

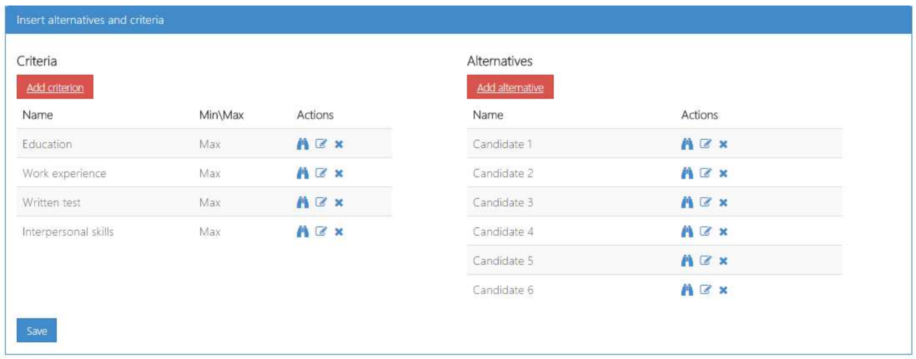


Figure 6: Insert fuzzy alternatives and criteria

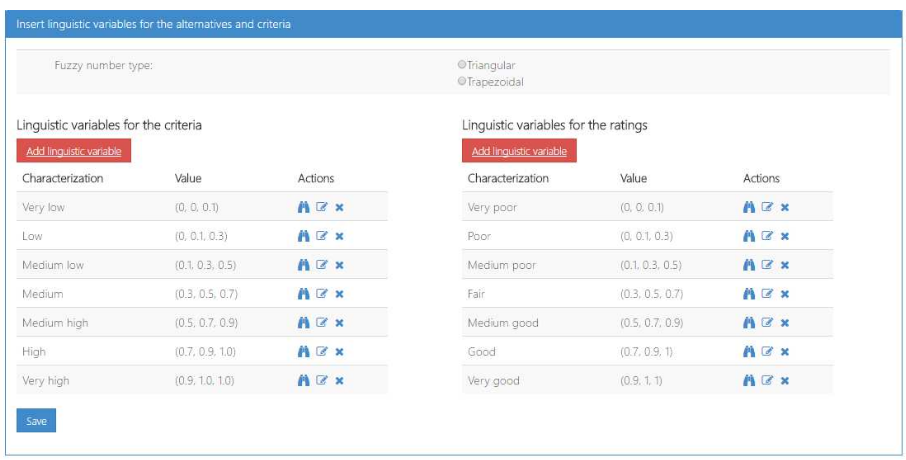


Figure 7: Insert linguistic variables for the alternatives and criteria

6. Illustrative Example

In this illustrative example, we use the proposed DSS to select the most suitable employee. We formed a group of three experts in order to make the best decision thus, we use fuzzy TOPSIS and VIKOR. There are six candidates, each

Figure 8: Select fuzzy algorithms and parameters

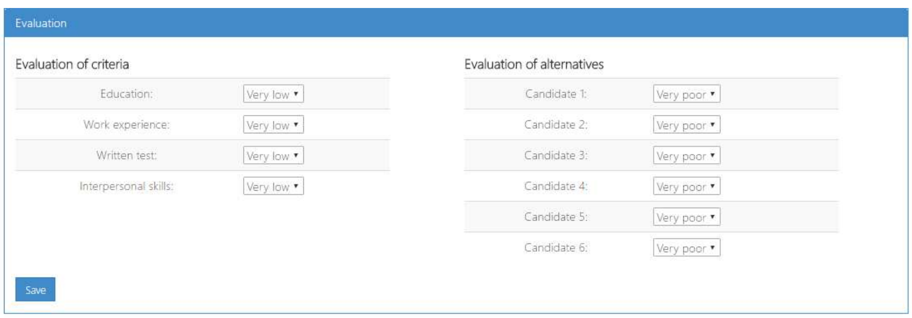


Figure 9: Evaluation

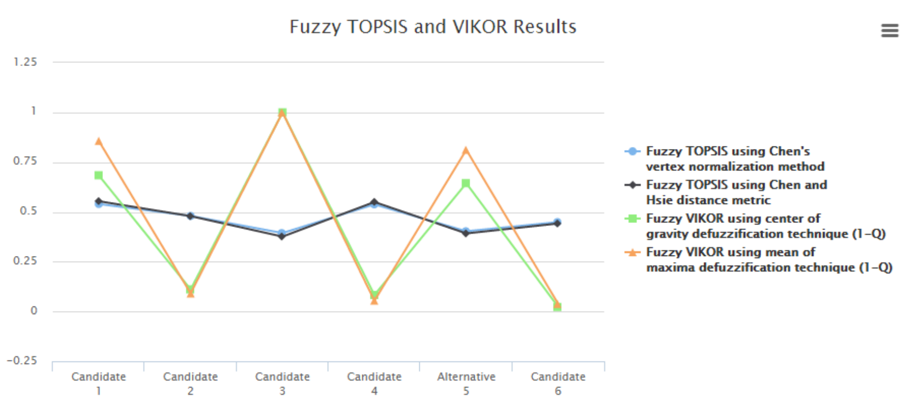


Figure 10: Graphical results

		Alternatives						Ranking
		C1	C2	C3	C4	C5	C6	
Fuzzy TOPSIS using Chen's vertex normalization method		0.607	0.558	0.637	0.515	0.498	0.526	C3, C1, C2, C6, C4, C5
Fuzzy TOPSIS using Chen and Hsie distance metric		0.629	0.573	0.663	0.528	0.506	0.538	C3, C1, C2, C6, C4, C5
Fuzzy VIKOR using center of gravity defuzzification technique	S	0.680	1.684	0.176	1.907	1.014	2.114	C3, C1, C5, C2, C4, C6
	R	0.433	0.900	0.156	0.856	0.364	0.866	C3, C5, C1, C4, C6, C2
	Q	0.316	0.889	0.000	0.917	0.356	0.977	C3, C1, C5, C2, C4, C6
Fuzzy VIKOR using mean of maxima defuzzification technique	S	0.387	1.695	0.144	1.869	0.597	2.038	C3, C1, C5, C2, C4, C6
	R	0.236	0.950	0.102	0.934	0.218	0.885	C3, C5, C1, C6, C4, C2
	Q	0.143	0.909	0.000	0.946	0.188	0.962	C3, C1, C5, C2, C4, C6

Figure 11: Tabular results

one of them evaluated according to the following four criteria: (i) education, (ii) work experience, (iii) written test, and (iv) interpersonal skills. The importance weights of the criteria and the ratings are considered as linguistic variables expressed in positive triangular fuzzy numbers, as shown in Table 3; they are also considered to be evaluated by decision makers that are experts on the field. The evaluations of three decision makers are in Tables 4 and 5.

Table 3: Linguistic variables for the criteria and the ratings

Linguistic variables for the importance weight of each criterion		Linguistic variables for the ratings	
Very low (VL)	(0, 0, 0.1)	Very poor (VP)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)	Poor (P)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)	Medium poor (MP)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)	Fair (F)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)	Medium good (MG)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1.0)	Good (G)	(0.7, 0.9, 1)
Very high (VH)	(0.9, 1.0, 1.0)	Very good (VG)	(0.9, 1, 1)

Table 4: The importance weight of the criteria for each decision maker

	D_1	D_2	D_3
Education	MH	M	H
Work experience	VH	H	VH
Written test	VH	VH	VH
Interpersonal skills	M	M	MH

Table 5: The ratings of the six candidates by the three decision makers for the four criteria

Criteria	Candidates	Decision makers			Criteria	Candidates	Decision makers		
		D_1	D_2	D_3			D_1	D_2	D_3
Education	Candidate 1	G	G	MG	Written test	Candidate 1	VG	VG	VG
	Candidate 2	G	F	G		Candidate 2	VG	VG	VG
	Candidate 3	VG	VG	VG		Candidate 3	VG	VG	VG
	Candidate 4	F	G	G		Candidate 4	G	G	G
	Candidate 5	MG	G	G		Candidate 5	G	VG	G
	Candidate 6	G	G	G		Candidate 6	G	G	F
Work experience	Candidate 1	VG	VG	VG	Interpersonal skills	Candidate 1	F	MP	MG
	Candidate 2	F	F	F		Candidate 2	VG	G	VG
	Candidate 3	G	G	F		Candidate 3	G	VG	VG
	Candidate 4	MP	MP	F		Candidate 4	VG	VG	VG
	Candidate 5	F	F	MP		Candidate 5	F	F	MG
	Candidate 6	F	F	G		Candidate 6	MG	F	G

The final results are shown in Figures 10 and 11 and Table 6. Both fuzzy TOPSIS combinations produce the same ranking. According to TOPSIS, the most suitable candidate is C3. On the other hand, both fuzzy VIKOR combinations generate the same ranking (considering only the measure Q) but a different one compared to fuzzy TOPSIS combinations. However, the two top-ranked candidates are the same on all four variants. A difference between the two fuzzy VIKOR combinations is that the one using the center of gravity defuzzification technique proposes as a compromise solution alternative C3, while the one using the mean of maxima defuzzification technique proposes alternatives C3 and C1.

Table 6: Results

Alternatives								**Ranking**
		C1	**C2**	**C3**	**C4**	**C5**	**C6**	
Fuzzy TOPSIS using Chen's vertex normalization method		0.607	0.558	0.637	0.515	0.498	0.526	C3, C1, C2, C6, C4, C5
Fuzzy TOPSIS using Chen and Hsie distance metric		0.629	0.573	0.663	0.528	0.506	0.538	C3, C1, C2, C6, C4, C5
Fuzzy VIKOR using center of	**S**	0.68	1.684	0.176	1.907	1.014	2.114	C3, C1, C5, C2, C4, C6
gravity defuzzification technique	**R**	0.433	0.9	0.156	0.856	0.364	0.866	C3, C5, C1, C4, C6, C2
	Q	0.316	0.889	0	0.917	0.356	0.977	C3, C1, C5, C2, C4, C6
Fuzzy VIKOR using mean of	**S**	0.387	1.695	0.144	1.869	0.597	2.038	C3, C1, C5, C2, C4, C6
maxima defuzzification technique	**R**	0.236	0.95	0.102	0.934	0.218	0.885	C3, C5, C1, C6, C4, C2
	Q	0.143	0.909	0	0.946	0.188	0.962	C3, C1, C5, C2, C4, C6

7. Conclusions

A significant problem researchers face when dealing with a multicriteria decision making problem is choosing the most suitable method for their problem. Even when finding a single MADM method that is appropriate to solve a specific problem, there may be many variants of this method that can be used. Many of these MADM methods and variants may produce different results. Hence, many researchers apply different MADM methods and compare the corresponding rankings. That way they have in their possession different scenarios and can select the one that is most suitable to their needs. If the results obtained by using different methods are similar, this fact may be considered as good indication that the proposed solution is optimal. In the opposite case, additional analysis of the criteria and their ranking is advised.

In this paper, we presented a DSS that enables decision makers use different methods and compare graphically the associate solutions. We reviewed the TOPSIS and VIKOR methods both in a fuzzy and in a nonfuzzy environment. Without trying to propose which method is best, we give the opportunity to decision makers to experiment with different methods and variations and decide which one fits their problem information. Reviewers can study various scenarios and fine-tune their models in order to find more appropriate solutions.

In that context, we implemented in the proposed DSS four MADM methodologies: (i) TOPSIS, (ii) VIKOR, (iii) fuzzy TOPSIS, and (iv) fuzzy VIKOR. For each of this method we also implemented different techniques at each step, resulting to ten normalization techniques, three methods for the calculation of the ideal and anti-ideal solutions, fifteen distance metrics, five fuzzy distance metrics, and eight defuzzification techniques. To the best of our knowledge, this is the first that most of these methods are being used in TOPIS and VIKOR. The proposed system can be used both in single and in group decision making problems.

We also presented an illustrative example of selecting the most suitable employee using a group of three experts. We applied fuzzy TOPSIS and VIKOR and build different scenarios using several techniques. In the specific example, although there were variations in the final ranking among different methods, all methods found the same two top-ranked alternatives. Hence, that is a good indication that these two alternatives should be ranked high. The proposed DSS gives the ability to decision makers to perform such scenarios and compare the results among many different MADM methods and/or techniques used in each MADM method. In addition, we presented the advantages that a web-based solution has when decisions are made in a group decision making environment. The DSS allows multiple decision makers collaborate without directly interacting with each other. The user-friendly interface of the DSS makes this procedure easy.

Finally, extensive comparisons in different application areas should be made in order to study the effect of using a different technique at a step of a specific MADM method, e.g., a normalization method for TOPSIS, a method for selecting the ideal and anti-ideal solutions for VIKOR, a distance metric in fuzzy TOPSIS, and a defuzzification technique in fuzzy VIKOR. In future work, we plan to incorporate in the proposed DSS other MADM methods, like PROMETHEE and AHP, and study the effect of using a different technique at a step of an MADM method. We also plan to include the revised Simos procedure for the evaluation of weights and address a problem that often arises in group decision making; the fact that many decision makers do not have the same expertise or experience that would allow them to safely reach a solution. This is why we plan to include a module allowing them to have different importance and as such have a different effect in the final result.

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