Heat Exchanger Circuitry Design by Decision Diagrams

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Abstract. The interconnection pattern between the tubes of a tube-fin heat exchanger, also referred to as its circuitry, has a significant impact on its performance. We can improve the performance of a heat exchanger by identifying optimized circuitry designs. This task is difficult because the number of possible circuitries is very large, and because the dependence of the heat exchanger performance on the input (i.e., a given circuitry) is highly discontinuous and nonlinear. In this paper, we propose a novel decision diagram formulation and present computational results using the mixed integer programming solver CPLEX. The results show that the proposed approach has a favorable scaling with respect to number of tubes in the heat exchanger size and produces configurations with 9% higher heat capacity, on average, than the baseline configuration.

Keywords: Optimization · Decision diagram · Heat exchanger design · Refrigerant circuitry · Heat capacity.

1 Introduction

Heat exchanger performance is important in many systems, ranging from the heating and air-conditioning systems that are widely used in residential and commercial applications, to plant operation for process industries. A variety of shapes and configurations can be used for the constituent components of the heat exchanger, depending on its application [2]. The most common configuration in heating and air-conditioning is the crossflow fin-and-tube type. In this type, a refrigerant flows through a set of pipes and moist air flows across a possibly enhanced surface on the other side of the pipe, allowing thermal energy to be transferred between the air and the refrigerant.

Heat exchanger performance can be improved according to a number of different metrics; these typically include maximization of heating or cooling capacity, size reduction, component material reduction, manufacturing cost reduction, reduction of pumping power, or a combination of these metrics. While the concept of many of these metrics is reasonably straightforward (e.g., size reduction and manufacturing cost reduction), the heat capacity is influenced by various parameters (like the geometry of the heat exchanger and the inlet conditions) and the dependence of the heat exchanger performance on these parameters tends to be highly discontinuous and nonlinear.

The *circuitry* determines the sequence of tubes through which the refrigerant flows and has a significant influence on the thermal performance of the heat exchanger. As heat exchangers for contemporary air-source heat pumps often have between 60 and 200 tubes, design engineers are faced with a very large number of potential circuitry choices that must be evaluated to identify a suitable design that meets performance and manufacturing specifications. Current design processes typically involve the manual choice of the configuration based upon expert knowledge and the results of an enumerated set of simulations. This task is inherently challenging, and does not guarantee that a manually found configuration will be optimal.

Systematic optimization of heat exchangers has been a long-standing research topic [3, 4]. Circuitry optimization is a particularly challenging task because: (i) the search space is enormous, making exhaustive search algorithms impractical for large numbers of tubes, and (ii) there is a highly discontinuous and nonlinear relationship between the circuitry design and the heat exchanger performance. Many researchers [6–9] have studied the effect of improving the refrigerant circuitry, and have concluded that circuitry optimization is often more convenient and less expensive than optimizing the geometry of the fins and tubes. Moreover, it has also been found that the optimal circuitry design for a specific heat exchanger is different from that of other heat exchangers [10].

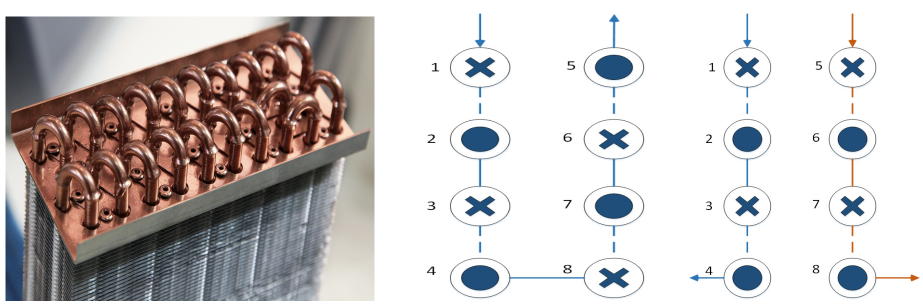
A variety of methods have thus been proposed to tackle the circuitry optimization problem [11–17]. These methods generally require either a significant amount of time to find the optimal circuitry design or generate a circuitry that is difficult to manufacture. In [1], we presented a binary constrained formulation for the heat exchanger circuitry optimization problem that generates circuitry designs without requiring extensive domain knowledge. Derivative-free optimization algorithms were applied to optimize heat exchanger performance and constraint programming methods were used to verify the results for small heat exchangers.

In this paper, we extend our work in [1] by providing a novel *relaxed* decision diagram formulation for the heat exchanger circuitry optimization problem. Decision diagrams have played a variety of roles in discrete optimization [18–31]. In a number of applications the decision diagram formulation has vastly outperformed existing formulations [20, 23, 29, 27]. Our new formulation produces smaller optimization instances and is able to find optimized circuitry configurations on heat exchangers with 128 tubes. In contrast, the approach in [1] could only optimize coils up to 36 tubes.

The remainder of this paper is organized as follows. In Section 2, we present circuitry design principles of a heat exchanger. Section 3 describes the proposed formulation for optimizing the performance of heat exchangers. Section 4 presents the computational experiments on finding the best circuitry arrangements on various heat exchangers and also provides a discussion on the advantages of the formulation. Conclusions from the research are presented in Section 5.

2 Heat exchanger circuitry

In this paper, we assume that all geometric and inlet parameters are predefined. As described in the introduction, the main problem of interest is to determine the circuitry configuration that optimizes the heat exchanger performance. This configuration, which is typically realized during the manufacturing process, includes both the circuitry design and the identification of the inlet and outlet tubes. Figure 1(a) is an illustration of the circuitry for a representative heat exchanger. The manufacturing process for fin-tube heat exchangers typically proceeds by first stacking layers of aluminum fins together that contain preformed holes, and then press-fitting copper tubes into each set of aligned holes. The copper tubes are often pre-bent into a U shape before insertion, so that two holes are filled at one time. After all of the tubes are inserted into the set of aluminum fins, the heat exchanger is flipped over and the other ends of the copper tubes are connected in the desired circuitry pattern. Figures 1(b) and 1(c) illustrate circuitry



(a) (b) (c)

Fig. 1: (a) Illustration of heat exchanger (Image licensed from S. S. Popov/Shutterstock.com). (b) and (c) are examples of valid circuitry configurations with one and two circuits, respectively.

configurations for a heat exchanger of eight tubes. A crossed sign inside a circle indicates that the refrigerant flows into the page, while a dotted sign indicates that the refrigerant flows out of the page. There are two types of connections: (i) a connection at the far end of the tubes, and (ii) a connection at the front end of the tubes. Therefore, a dotted line between two tubes represents a connection on the far end (pre-bent tube), while a solid line represents a connection on the front end of the tubes. In this example, the pairs of tubes 1-2, 3-4, 5-6 and 7-8 are the pre-connected tubes (i.e. tubes with bends on the far end of the coil). In

Figure 1(b), the inlet stream is connected tube1 and outlet stream is connected to tube 5. In Figure 1(c), tubes 1 and 5 are connected to inlet streams, while tubes 4 and 8 are connected to outlet streams. A given *circuit* is a set of pipes through which the refrigerant flows from inlet to outlet. Figures 1(b) and 1(c) depict circuitry configuration with one and two circuits respectively.

A set of realistic manufacturing constraints are imposed on the connections of the tubes: (i) adjacent pairs of tubes in each column, starting with the bottom tube, are always connected (this constraint is imposed by the manufacturing process since one set of bends on the far end are applied to the tubes before they are inserted into the fins), (ii) the connections on the far end cannot be across rows unless they are at the edge of the coil, (iii) plugged tubes, i.e., tubes without connections, are not allowed, (iv) inlets and outlets must always be located at the near end, and (v) merges and splits are not allowed.

3 Decision diagram formulation

In [1], we proposed a new approach for formulating the refrigerant circuitry design problem. We formulated the problem as a binary constrained optimization problem with a black-box objective function and we applied derivative-free optimization algorithms to solve this problem. Each connection was represented using a binary variable and cycles were excluded by adding inequality constraints. As a result, the constraint matrix was dense. In this paper, we propose a new formulation based on decision diagrams.

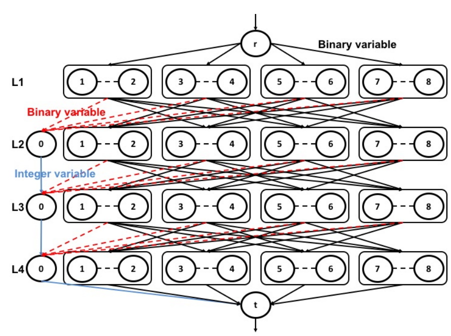


Fig. 2: Decision diagram formulation.

The main idea is that pre-connected tubes (i.e., tubes with bends on the far end of the coil) are treated as single entities which we call *super-nodes*. Based on the manufacturing constraint outlined previously, the heat-exchanger circuitry configuration can be defined as: (a) as a collection of paths involving super-nodes where each super-node occurs only once in a path; (b) paths cover all super-nodes; and (c) paths are super-node disjoint. We propose a *relaxed* decision diagram to represent the set of all heat exchanger configurations. The said diagram is *relaxed* since the requirements (a) and (c) are not modeled in the diagram; they are not ignored though, rather will be enforced by additional constraints later on. Figure 2 shows such a relaxed decision diagram for a heat exchanger with eight tubes.

Let us assume that we have n tubes. The number of layers in the decision diagram is equal to $N = \frac{n}{2}$. The layers are indexed sequentially and every layer consists of the set of super-nodes. In addition, a 0-node is introduced into layers with index 2 and above. The 0-node represents the end of a circuit. Arcs are drawn between the nodes (collection of super-nodes and 0-node) of two successive layers. Root and terminal nodes are introduced that respectively connect to the first and last layers in the diagram. In this representation, a path from the root to terminal can repeat super-nodes (refer to Figure 2). For example, the path $(\mathsf{r}, 1{-}2, 3{-}4, \mathsf{0}, \mathsf{0}, \mathsf{t})$ is path satisfying (a) while the path $(\mathsf{r}, 1{-}2, 3{-}4, 1{-}2, 3{-}4, \mathsf{t})$ is a path that does not satisfy (a). However, we impose additional constraints that ensure that we identify configurations satisfying the requirements (a)-(c). The constraints ensure that the identified path is indeed a *circuit*.

Before presenting the mixed integer programming model derived from this decision diagram formulation, we introduce the following notation:

- N: the number of layers in the decision diagram
- L_i: represents the i-th layer in the decision diagram, where $i = 1, \dots, \mathsf{N}$
- s: super-nodes (not including 0-node)
- S: set of super-nodes
- r, t: the root and terminal nodes in the decision diagram
- (s, i) or $(\mathsf{0}, i)$: node in layer i of decision diagram
- a: arcs in the decision diagram
- $\mathsf{head}(\mathsf{a})$ $(\mathsf{tail}(\mathsf{a}))$: starting (ending) node of the arc in the decision diagram
- $\mathsf{A}^{in}_{\mathsf{s},i}$ $(\mathsf{A}^{out}_{\mathsf{s},i})$: set of input arcs to (output arcs from) super-node s in L_i
- $\mathsf{A}^{in}_{\mathsf{0},i}$ $(\mathsf{A}^{out}_{\mathsf{0},i})$: set of input arcs to (output arcs from) $\mathsf{0}$ in L_i
- $x_\mathsf{a} \in \{0,1\}$ for $\mathsf{a} \in \mathsf{A}(x) := \bigcup_{i=1}^{\mathsf{N}} \bigcup_{\mathsf{s}\in\mathsf{S}} \left(\mathsf{A}^{in}_{\mathsf{s},i} \cup \mathsf{A}^{out}_{\mathsf{s},i}\right)$: binary variables encoding flow on the arcs between $\mathsf{s}, \mathsf{s}' \in \mathsf{S}$ and flow on arcs between $\mathsf{s} \in \mathsf{S}$ and $\mathsf{0}$
- $z_\mathsf{a} \in \{0,1,\dots\}$ for $\mathsf{a} \in \mathsf{A}(z) := \bigcup_{i=2}^{\mathsf{N}} \mathsf{A}^{out}_{\mathsf{0},i}$: integer variables encoding flow on the arcs between $\mathsf{0}$ in successive layers
- C_{lb}: the minimum number of circuits
- C_{ub}: the maximum number of circuits

Therefore, the mixed integer programming model derived from the decision diagram formulation can be expressed as:

$$\max \quad Q(x,z) \left(\text{ or } \frac{Q(x,z)}{\Delta P(x,z)} \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{\mathsf{a}\in\mathsf{A}^{in}_{\mathsf{s},i}} x_\mathsf{a} = \sum_{\mathsf{a}\in\mathsf{A}^{out}_{\mathsf{s},i}} x_\mathsf{a}, \qquad \forall\, \mathsf{s}\in\mathsf{S}, i \in \{1,\dots,\mathsf{N}\} \tag{2}$$

$$\sum_{\mathsf{a}\in\mathsf{A}^{in}_{\mathsf{0},i}:\mathsf{tail}(\mathsf{a})\in\mathsf{S}} x_\mathsf{a} + \sum_{\mathsf{a}\in\mathsf{A}^{in}_{\mathsf{0},i}:\mathsf{tail}(\mathsf{a})=\mathsf{0}} z_\mathsf{a} = \sum_{\mathsf{a}\in\mathsf{A}^{out}_{\mathsf{0},i}} z_\mathsf{a}, \; \forall\, i = 2,\dots,\mathsf{N} \tag{3}$$

$$\sum_{i=1}^{N} \sum_{\mathsf{a}\in\mathsf{A}^{in}_{\mathsf{s},i}} x_\mathsf{a} = 1, \qquad \forall\, \mathsf{s}\in\mathsf{S} \tag{4}$$

$$C_{lb} \le \sum_{\mathsf{a}\in\bigcup_{\mathsf{s}\in\mathsf{S}} \mathsf{A}^{in}_{\mathsf{s},1}} x_\mathsf{a} \le C_{ub}, \tag{5}$$

$$x_\mathsf{a} \in \{0,1\}\,, \;\; \mathsf{a}\in\mathsf{A}(x), \;\; z_{\mathsf{a}'} \in \mathbb{Z}, \qquad \forall\, \mathsf{a}' \in \mathsf{A}(z). \tag{6}$$

where $Q(x, z)$ is the heat capacity related to the solution vectors x and z, and ΔP is the pressure difference across the heat exchanger.

Two targets of the refrigerant circuitry optimization are considered in this work: (i) maximization of the heat capacity ($Q(x, z)$), and (ii) maximization of the ratio of the heat capacity to the pressure difference across the heat exchanger ($Q(x, z)/\Delta P(x, z)$). Constraint (2) is the flow balance for the super-nodes in all different levels, while constraint (3) is the flow balance for the 0-nodes. Note that $\{\mathsf{a} \in \mathsf{A}_{0,2}^{in} \,|\, \mathsf{tail}(\mathsf{a}) = 0\} = \emptyset$. Constraint (4) is imposed for each super-node s and invalidates any repetition of super-nodes, so there can be no cycles. Constraint (5) sets a limit on the number of circuits in the circuitry configuration.

The total number of variables in the decision diagram formulation is equal to $|\mathsf{S}|^3 - |\mathsf{S}|^2 + 3\,|\mathsf{S}| - 1$ and the total number of constraints is equal to $|\mathsf{S}|^2 + 2\,|\mathsf{S}| + 2$. Table 1 shows the superiority of the proposed formulation compared to the one proposed in [1]. The formulation in [1] is memory bound, i.e., it requires an exponentially increasing amount of memory for only a constant increase in problem size. Moreover, the constraint matrix in [1] is dense, while the constraint matrix in the proposed formulation is sparse. For example, the constraint matrix for a heat exchanger with 40 tubes of the formulation in [1] needs $\sim$ 6GB of memory, while the constraint matrix of the proposed formulation needs only $\sim$ 300KB.

Table 1: Reduction in the problem size of the proposed decision diagram formulation compared to the formulation proposed in [1]

# of tubes	Problem size of the formulation proposed in [1]	Problem size of the proposed formulation	Reduction in problem size (%)
16	263×120	471×82	-22%
24	$4,107 \times 276$	$1,619 \times 170$	76%
32	$65,551 \times 496$	$3,887 \times 290$	97%
40	$1,048,595 \times 780$	$7,659 \times 442$	100%
128	-	$258,239 \times 4,226$	-

We provide a brief discussion on the advantages of the decision diagram based representation. The width and the depth (number of layers) of the diagram grows linearly in the number of tubes. This can be quite prohibitive in that it leads to a very dense formulation for heat exchangers with a large number of tubes. However, operational and manufacturing constraints help to alleviate this complexity. From practical operational considerations, it is not desirable to have long circuits since they incur large pressure drops and increased costs (pump power) to flow the refrigerant. Hence, from pressure drop considerations it is desirable to limit the depth of the diagram and this can be easily accomplished by truncating the diagram. From manufacturing considerations it is not desirable to allow connections between all pairs of tubes. This can also be accomplished by eliminating arcs between pairs of super-nodes for which a connection is forbidden.

This results in a much sparser diagram and an associated integer program which is smaller in size. We will explore these aspects in a future work.

4 Computational results

In order to validate the proposed model, we performed a computational study with the aim of optimizing the heat capacity ($Q(x,z)$) and the ratio of the heat capacity to the pressure difference ($Q(x,z)/\Delta P(x,z)$) across the heat exchanger. The analytical form of $Q(x,z)$ and $Q(x,z)/\Delta P(x,z)$ as a function of x,z is typically not available and hence, the optimization problem in (1)-(6) cannot be solved by mixed integer programming solvers such as CPLEX. The quantities $Q(x,z)$ and $Q(x,z)/\Delta P(x,z)$ can only be obtained by specifying a particular circuitry configuration defined by x,z as input to a heat exchanger simulation program such as CoilDesigner [32]. CoilDesigner is a steady-state simulation and design tool for air to refrigerant heat exchangers, to simulate the performance of different refrigerant circuitry designs.

We replaced the objective in (1) by a constant and applied the mixed integer programming solver CPLEX on (2)-(6) to produce $2,500$ feasible circuitry configurations. In order to achieve that, we used the appropriate CPLEX parameters to create a diverse solution pool of $2,500$ feasible solutions for this problem (parameters: PopulateLim, SolnPoolCapacity, and SolnPoolReplace). Then, we evaluated all the feasible configurations using CoilDesigner. We created a test suite with seven circuitry architectures with a varying number of tubes. The structural parameters and work conditions for all instances are the same; the only difference between the test cases is in the number of tubes per row, ranging from 2 to 64 that correspond in heat exchangers with 4 to 128 tubes.

Tables 2 and 3 present the results of the optimization of the two objective functions, $Q(x,z)$ and $Q(x,z)/\Delta P(x,z)$, respectively. We compare the optimized results generated by the proposed approach with the results in [1] and the results obtained using a baseline configuration, which includes two circuits: one that connects all tubes in the first column of the coil and one that connects all tubes in the second column of the coil. The inlet tubes of the baseline configuration are the tubes in the first row and the outlet tubes are the tubes in the last row. The baseline configuration is a heat exchanger design that is typically used in practice today. The results show that the proposed approach can generate optimized configurations in a short amount of time. On average, the proposed approach produces configurations with 4% and 9% higher heat capacity than the approach in [1] and the baseline configuration, respectively. In addition, the proposed approach produces configurations with 90% and $8,826\%$ higher ratio of the heat capacity to the pressure difference than the approach in [1] and the baseline configuration, respectively. It is worth noting that the formulation in [1] can only be used to solve problems with up to 36 coils, while the present formulation can be used to solve much larger problems. Therefore, the current approach not only produces better results than the approach in [1] but it can also solve much larger problems. In addition, the current approach needs an order of magnitude less execution time than the approach proposed in [1].

The limit of function evaluations $(2,500)$ prevents the proposed approach from finding even better results for very large coils. When optimizing $Q(x, z)$, the best circuitry configurations include a small number of long circuits and the circuits usually contain at least one connection between tubes across columns. On the other hand, when optimizing $Q(x, z)/\Delta P(x, z)$, the best circuitry configurations include many circuits that are not very long and most of the connections on these circuits are between adjacent tubes. This was expected since longer circuits incur more pressure drop.

Table 2: Computational results for Q optimization

# of tubes	Baseline Q	Optimized Q	Optimized Q in [1]	Improvement over baseline (%)	Improvement over [1] (%)
4	1,388	1,754	1,754	26%	0%
8	1,884	2,189	2,017	16%	9%
16	2,179	2,391	2,230	10%	7%
24	2,249	2,353	2,294	5%	3%
32	2,234	2,269	2,244	2%	1%
40	2,154	2,255	-	5%	-
128	9,694	9,790	-	1%	-

Table 3: Computational results for $Q/\Delta P$ optimization

# of tubes	Baseline $\frac{Q}{\Delta P}$	Optimized $\frac{Q}{\Delta P}$	Optimized $\frac{Q}{\Delta P}$ in [1]	Improvement over baseline (%)	Improvement over [1] (%)
4	3,727	3,727	3,727	0%	0%
8	2,640	14,664	12,464	455%	18%
16	1,668	51,219	32,985	2,971%	55%
24	1,162	110,289	47,865	9,391%	130%
32	854	156,181	45,292	18,188%	245%
40	632	193,654	-	30,541%	-
128	448	1,518	-	239%	-

5 Conclusions

The performance of a heat exchanger can be significantly improved by optimizing its circuitry configuration. Design engineers currently select the circuitry design based on their domain knowledge and some simulations. However, the design of an optimized circuitry is difficult and needs a systematic approach to be used. In this paper, we extended our work in [1] and proposed a novel decision diagram formulation for the circuitry optimization problem. The generated mixed integer programming problem is much smaller than the problem derived from the formulation in [1] and leads us to optimize coils with a very large number of tubes. We applied CPLEX to generate feasible configurations for seven different heat exchangers and we evaluated them using CoilDesigner. The results show that the proposed formulation can improve the baseline configuration by 9% for the heat capacity and by $8,826\%$ for the ratio of the heat capacity to the pressure difference than the baseline configuration. Finally, the proposed approach produces on average 4% higher heat capacity and 90% higher ratio of the heat capacity to the pressure difference than the approach in [1].

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